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Session - 2024 - 25

Sub:- Strength of Material (3rd sem)

Prepared by:-
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Unit-I

Direct stresses and strain in components.

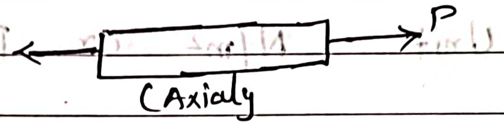
<u>Load</u>:- A load may be defined as a force tending to effect and produce deformation, stresses or ~~dif~~ displacement in the structure.

~~Lod~~ Load may be classified by number of way

~~(A) (i) tensile load (ii)~~

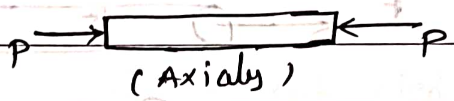
~~(A) (i) dead load (ii) fluctuating load~~

(A) (i) tensile load:-



P = tensile load

(ii) compressive load:-



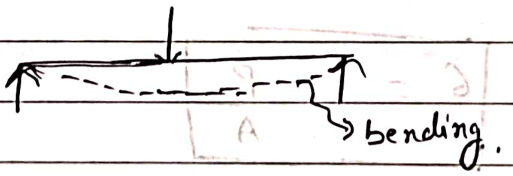
P = compressive load.

(iii) Shearing load:-

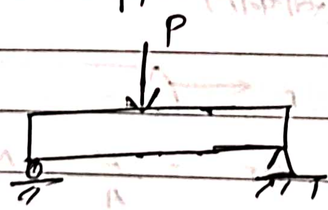
P = Shearing load

(iv) Torsilonal load (twisting load):-

(v) Bending load:-

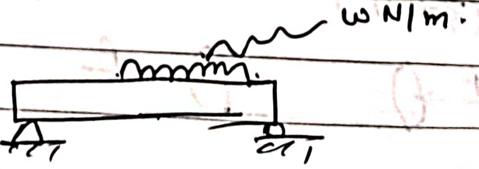


B. (i) <u>Point load</u>:- load, which is applied in small area or in single point.



P = point load
(N) → unit Newton.

(ii) <u>Distributed load</u>:- load, which is applied over a considerable length and area.



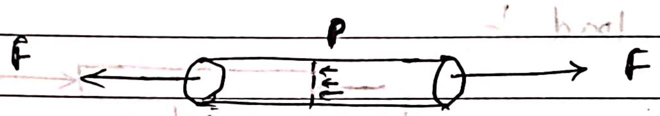
w = uniformaly distributed load.

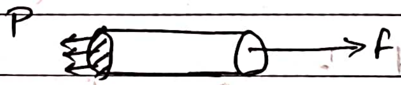
N/m → unit

Stress :- (प्रतिबल) :- When a material is ~~sty~~ subjected to an external force, a resisting force is setup within the material. this internal resistance Force per unit area is Known as stress.

किसी वस्तु के ईकाई क्षेत्रफल पर कार्य करने वाले आन्तरिक प्रतिरोधी बल को प्रतिबल ~~कह~~ कहते है।

Unit - N/m^2 or Pascal (Pa).





& F = P

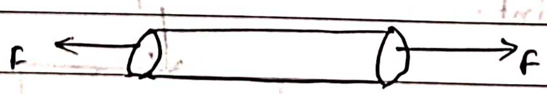
External force = Internal Resistance

$\therefore$ stress = $\frac{\text{internal resistance}}{\text{Area}}$ = $\frac{\text{आंतरिक ~~प्रति~~ प्रतिरोधी बल}}{\text{क्षेत्रफल}}$

$$\therefore \boxed{\sigma = \frac{P}{A}}$$

Types of stresses :-

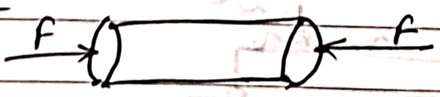
(a) Tensile stress :- (खींचना)



$\therefore \sigma_T = \frac{F}{A}$ N/m^2

σ_T = tensile ~~force~~ stress
F = tensile force
A = ~~Are~~ Cross-section Area.

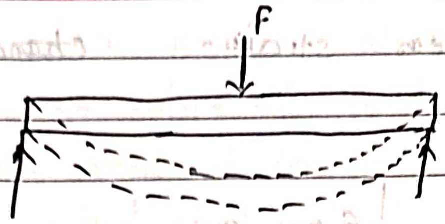
(ii) Compressive :- (दबाना)



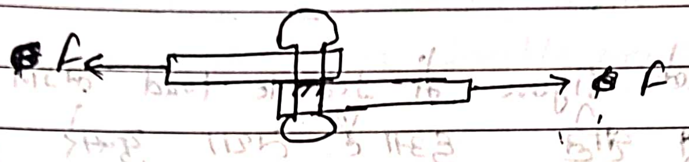
$\sigma_c = \frac{F}{A}$

σ_c = compressive stress
F = compressive force

(c) Bending stress :-



(d) Shear stress :-



(different line of Action.

τ = shear stress $= \frac{F}{A}$

A = Area (Parallel to force)

Strain :- (विकृति) When a body is subjected to some external force there is some change in dimensions of body. the ratio of change of dimension of the body to its orignal dimensions is known as strain.

वाह्य बलों के प्रभाव में Body में विरूपण होता है, या उसके आकार में परिवर्तन होता है। ~~परिवर्तन~~ इस परिवर्तन के मापन को ही विकृति कहते हैं।

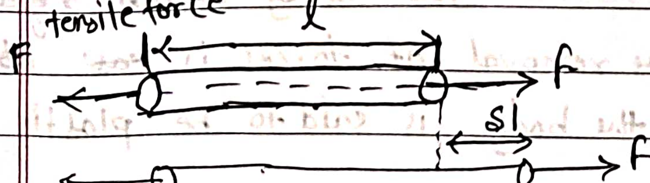
Strain = ε = $\frac{\text{Change in dimensions}}{\text{orignal dimensions}}$

Unit = strain की कोई Unit नहीं होती।

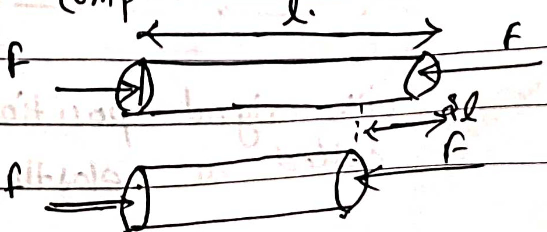
Types of strain :-

(i) Linear strain (Longitudinal strain) :-

tensile force



compressive force.



$\therefore$ linear strain = ~~chand~~ $\frac{\text{change in length}}{\text{orignal length.}}$

$$\boxed{\varepsilon \text{ or } e = \frac{\delta l}{l}}$$

Note:- पहले figure में tensile load लगाने के कारण लम्बाई में ~~परिवर्तन~~ वृद्धि हुआ है तथा दूसरे figure में compressi force लगाने के कारण लम्बाई में कमी आयी है।

(ii) Lateral strain:- may be defined as the ratio of change in diameter to orignal diameter.

$$\varepsilon_d \text{ or } e_d = \frac{\text{change in diameter}}{\text{orignal diameter}} = \frac{\delta d}{d}.$$

Note:- पहले figure में tensile force लगाने के कारण व्यास में कमी आयी है, तथा दूसरे figure में compressive force लगाने के कारण व्यास (diameter) में वृद्धि हुयी है।

(iii) Volumetric strain:- may be defined as the ratio of change in volume to orignal volume.

$$e_v = \frac{\text{change in volume}}{\text{orignal volume.}} = \frac{\delta v}{V.}$$

Modulus of elasticity:- or (Young's Modulus):- the body which regain it's orignal position on the removal of forces is called as elastic body. the body is said to be plastic if the strain exist even after the removal of external forces. there is always a limiting value of load uptowhich strain totally disappears on the removal of load - the stress corres-poing to this load is called elastic limit.

Robert Hooke discovered that within elastic limit stress is directly proportional to strain.

ie.

$$\text{stress} \propto \text{strain}$$

$$\frac{\text{Stress}}{\text{strain}} = \text{a } \cancel{\text{constt}} \text{ constant}$$

this constant termed as Modulus of elasticity (E).

$$\text{Stress} = E \times \text{Strain}$$

$$\boxed{\sigma = E \times e}$$

E = Modulus of elasticity or Young's modulus.

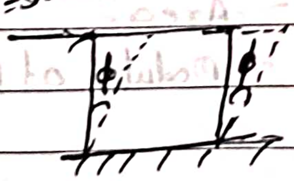
Unit - N/m^2 or Pa.

Modulus of rigidity :- (G or C) may be defined as the ratio of shear stress (τ) to shear strain (ϕ)*. and denoted by (G or C).

$$\text{Modulus of rigidity (G or C)} = \frac{\text{Shear Stress}}{\text{Shear strain}}$$

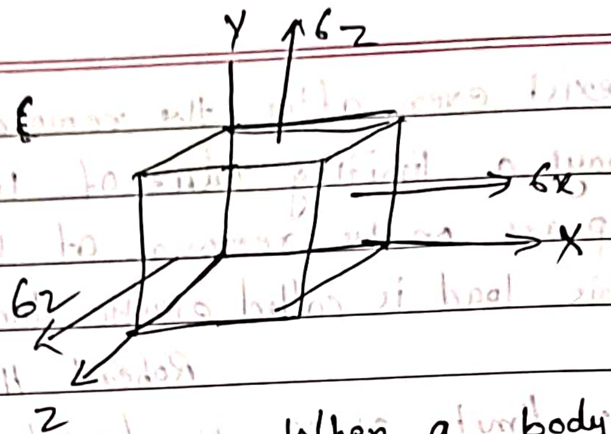
$$\boxed{G = \frac{\tau}{\phi}}$$

F = shear load.

* 

in case of shearing load, a shear strain will be produced which is measured by the angle through which the body distorted.

Bulk Modulus:- E



When a body is subjected to three mutually perpendicular stress of a equal intensity the ratio of direct stress to the corresponding volumetric strain is known as Bulk modulus.

$$\boxed{\text{Bulk modulus } (K) = \frac{\sigma}{e_v}}$$

Unit - N/m2

From Hook's law:-

$$\sigma \propto e$$

$$\sigma = E \times e$$

where

σ = stress = $\frac{P}{A}$ E = modulus of elasticity   e = strain.

put the values.

$$\frac{P}{A} = E \times \frac{\delta l}{l}$$

Imp.

$$\boxed{\delta l = \frac{P \times l}{A \times E}}$$

where:-

δl = change in length.

P = load.

A = Area.

E = Modulus of Elasticity.

Imp

Poission ratio:-

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Time Table 4th and 6th Semester

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& for 6th SEM- "MF 02" W.E.F 6th SEM Date 06/01/2025 W.E.F 4th SEM Date 03/02/25

2nd 11:30-12:30	3rd 12:30-01:30	01:30 TO 02:00	4th 02:00-03:00	5th 03:00-04:00	6th 04:00
FMM	MP		IMC(LAB)		

enue for 3rd SEM – "MF 01" & for 5th SEM- "MF

DAY	SEM	1st 10:30 -11:30	2nd 11:30-12:30
MON	3rd	BEE	SOM

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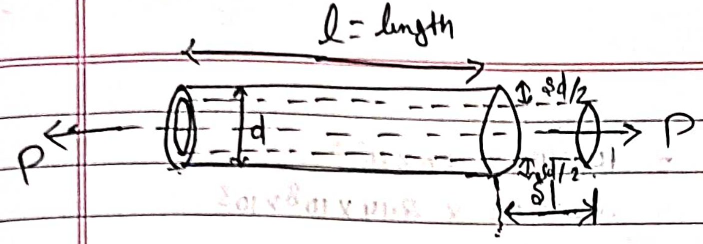


~~खला किसी कार~~

∴ linear strain = $\dfrac{\text{change in length}}{\text{orignal length.}}$

lateral strain = $\dfrac{\text{change in diamter}}{\text{orignal diameter.}}$

∴ poission ratio may be defined as the ration of lateral strain to linear strain. poission ratio is denoted by (μ) mu

$$\boxed{\text{poission ratio} = \dfrac{\text{lateral strain}}{\text{linear strain}}}$$

R.K.

Q:-1 A square steel rod 20m.m x 20m.m. in section is to carry an axia load (compressive) of 100kN. Calculate the shortening in length. of 50m.m.. $E = 2.14 \times 10^8$ kN/m².

Sol:- Given:- (इकट्ठा किये गये Data को S.I. Units में लिखें)

Area = A = 20 m.m. × 20 m.m. = .02m. × .02m.

P = 100 kN = 100×10^3 N (compressive)

l = 50m.m. = 50×10^{-3} m.

E = 2.14×10^8 kN/m² = $2.14 \times 10^8 \times 10^3$ N/m²

δl = ?

from the formula:-

$$\delta l = \frac{P \times l}{A \times E}$$ ~~(Pattha val~~

Putting the values :-

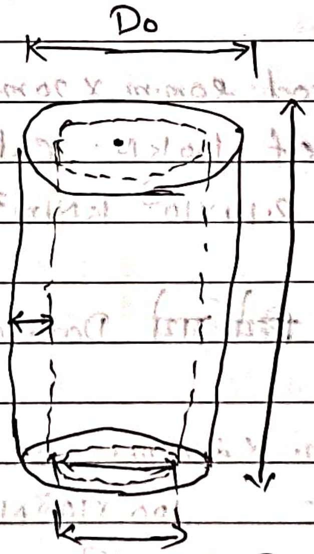
$$\delta l = \frac{100 \times 10^3 \times 50 \times 10^{-3}}{.02 \times .02 \times 2.14 \times 10^8 \times 10^3}$$

$\delta l = .0584 \times 10^{-3}$ m. **Answer.**

∵ As the given load is compressive in nature so the change in length i.e. shortening in length is $= .0584 \times 10^{-3}$

Q:2 A hollow cast-iron cylinder 4m. long, 300 m.m. outer diameter, and thickness of metal 50 m.m. is subjected to a central load on the top when standing straight. the stress produce is 75000 KN/m². Assume :- young's modulus = 1.5×10^8 KN/m².
find :- (i) magnitude of the load
(ii) longitudinal strain
(iii) total decrease in length.

Sol :- Given :-



$l = 4m$

$D_o = 300$ m.m. $= .3m$.

$t = 50$ m.m. $= .05m$.

∴ $D_i = D_o - 2t$
$= .3 - .05 \times 2$
$= .2m$.

Stress $= \sigma = 75000 \times 10^3$ N/m²

$E = 1.5 \times 10^8$ KN/m².
$= 1.5 \times 10^8 \times 10^3$ N/m²
$= 15 \times 10^{11}$ N/m².

$P = ?$

$\varepsilon = ?$

$\delta l = ?$

$\therefore$ σ = stress = $\frac{P}{A}$

$$75000 \times 10^3 = \frac{P}{\frac{\pi}{4} \times (D_o^2 - D_i^2)}$$

$\therefore P = 7500 \times 10^3 \times \frac{\pi}{4} \times (.3^2 - .2^2)$

$P = 2945.2$ KN. Ans

again stress = E × strain.

$\therefore$ strain = e = $\frac{\text{stress}}{E}$

$$= \frac{75000 \times 10^3}{1.5 \times 10^8 \times 10^3} = .0005 \text{ Ans}$$

$\therefore$ e = 0.0005

and $e = \frac{\delta l}{l}$ $\therefore$ $\frac{\delta l}{l} = .0005$

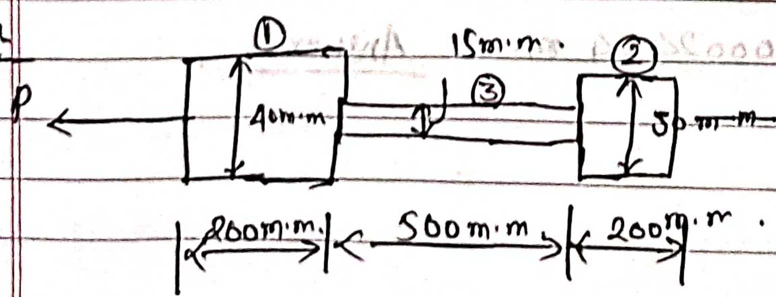
$\delta l = .0005 \times 4$

$\delta l = .002$ m. Ans

Q:-3 A steel bar is 900 m.m. long. it's two ends are 40m.m. and 50m.m. in diameter and the length of each rod is ~~200m.m~~ 200m.m. the middle portion of bar is 15m.m in diameter and 500m.m. long. is the bar is subjected to one axial tensile load of 15KN. find the total extension

E = 200 GN/m2   G → Giga.

Sol:



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Given:- $P = 15kN = 15 \times 10^3 N.$ $E = 200 \times 10^9 N/m^2.$

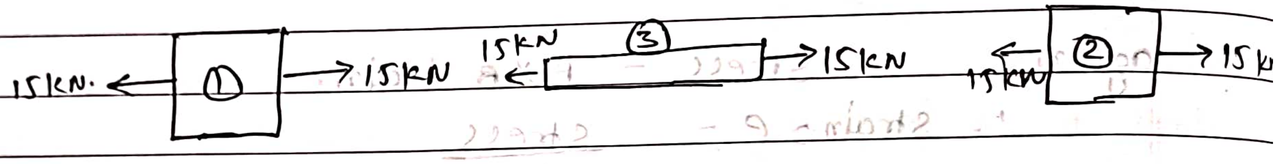
$D_1 = 40mm = .04m.$ $A_1 = \pi/4 \times .04^2 = .001256 m^2$

$D_2 = 30mm = .030m.$ $A_2 = \pi/4 \times .03^2 = \cancel{.0001767} \; .0007068 m^2$

$D_3 = 15mm = .015m.$ $A_3 = \pi/4 \times .015^2 = .0001767 m^2.$

$l_1 = .2m.$ $l_2 = .2m.$ $l_3 = .5m.$

free body daigram:-



$$\delta l_1 = \frac{P_1 \times l_1}{A_1 \times E_1} = \frac{15 \times 10^3 \times .2}{.001256 \times 200 \times 10^9} = 1.194 \times 10^{-5} m$$

$$\delta l_2 = \frac{P_2 \, l_2}{A_2 \, E_3} = \frac{15 \times 10^3 \times .2}{.0007068 \times 200 \times 10^9} = 2.122 \times 10^{-5} m.$$

$$\delta l_3 = \frac{P_3 \times l_3}{A_3 \times E_3} = \frac{15 \times 10^3 \times .5}{.0001767 \times 200 \times 10^9} = 2.122 \times 10^{-4} m.$$

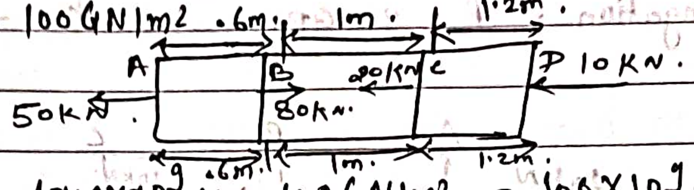
Total extension of bar:-

$$\delta l = \delta l_1 + \delta l_2 + \delta l_3$$

$$\delta l = \left[\frac{15 \times 10^3 \times .2}{.001256 \times 200 \times 10^9} + \frac{15 \times 10^3 \times .2}{.0007068 \times 200 \times 10^9} + \frac{15 \times 10^3 \times .5}{.000176 \times 200 \times 10^9} \right.$$

$\delta l = 0.0002454 m.$ Answer.

Q.1 A brass having cross-sectional Area $1000\,mm^2$ is subjected to axial force shown in fig. find the total elongation of bar. $E = 100\,GN/m^2$



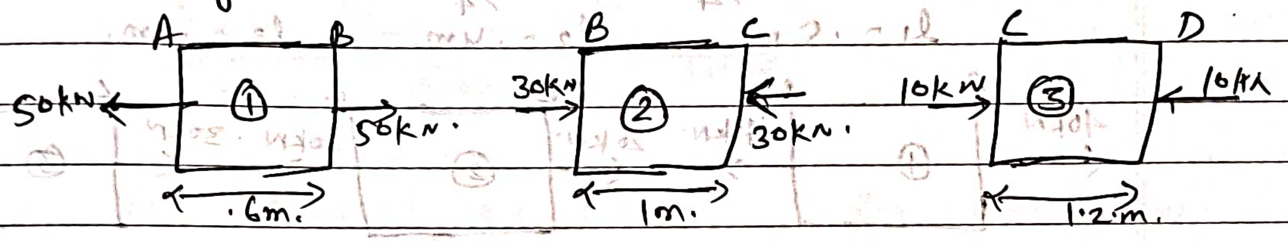
Sol: Given:-

$E = 100\,GN/m^2 = 100\times10^9\,N/m^2.$

$A = 1000\,mm^2$

$= 1000\times10^{-6}\,m^2 = 10^{-3}\,m^2.$

free body daigram:-



$$\delta l_1 = \frac{P_1\times l_1}{A_1\times E_1} \qquad \delta l_2 = \frac{P_2\times l_2}{A_2\times E_2} \qquad \delta l_3 = \frac{P_3\times l_3}{A_3\times E_3}.$$

Imp ∵ BC तथा CD body में force compressive है अर्थात दोनो Body में लम्बाई में कमी आयेगी इसलिए परिणामी लम्बाई में परिवर्तन-

$$\delta l_{net} = \delta l_1 - \delta l_2 - \delta l_3$$

$$\delta l_{net} = \frac{50\times10^3\times.6}{10^{-3}\times10^{11}} - \frac{30\times10^3\times1}{10^{-3}\times10^{11}} - \frac{10\times10^3\times1.2}{10^{-3}\times10^{11}}$$

$= -.00012\,m.$

$= -.12\,mm$ Answer

Imp (-) negative sign shows that बार के लम्बाई में .12mm. की कमी आयेगी।

$$\delta_{net} = \frac{P_1 l_1}{A_1 \times E} - \frac{P_2 l_2}{A_2 E} + \frac{P_3 l_3}{A_3 E}$$

$$= \frac{50\times10^3\times1}{600\times10^{-6}\times210\times10^9} - \frac{3000\times10^3\times1}{2400\times10^{-6}\times210\times10^9} + \frac{200\times10^3\times .6}{1200\times10^{-6}\times21}$$

$\delta_{net} = .278\times10^{-3}$ m. So the length of the bar will increase by $= .278\times10^{-3}$ m.

Stress - strain curve for Mild Steel & (Ductile) :-

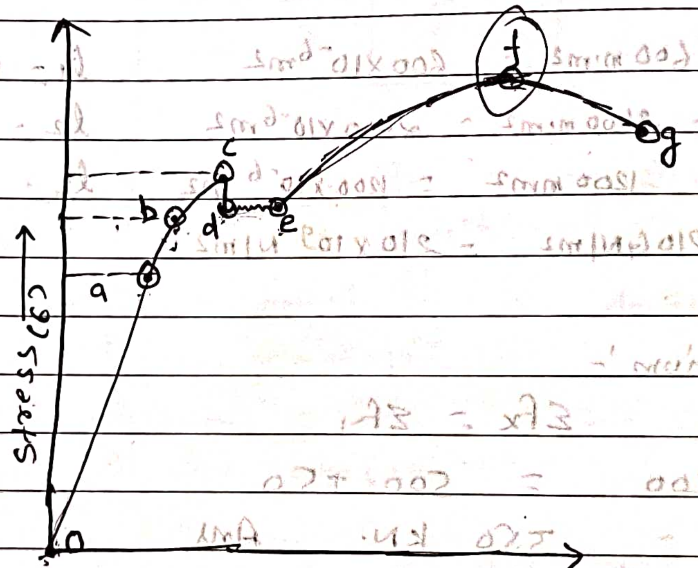


fig shows the typical stress - strain curve for ductile material (mild steel). initially curve starts from poin O where there is no stress and strain in specimen.

* up to point a, Hooke's law obeyed where stress is directly proportional strain therefore, Oa is a straight line and point a is called the limit of propornality and stress at point a is known as propotional limit stress.

* the portion of graph between a & b is not straight line. but upto point b material remain elastic i.e. (on removal of load material regains it's orignal shape). Point b is known as elastic limit point and the stress corresponding to that is called the elastic limit stress (σ_e).

In actual practice point a & b are so close to each other that it become difficult to differenciate between them.

* Beyond the point b material go to the plastic stage until upper yield point "c" is reached. At this point the cross-section area of material starts decreasing and stress decreases to a lower value to a point d, called lower yield point. corresponding point c, the stress is known as upper yield point stress σ_{yu}, and corresponding to point d, the stress is known as ~~upper~~ Lower yield point stress, σ_{yl}.

* At point d, the specimen elongate by a considerable amount without any increase in stress and upto point e. the portion de is called yielding of the material at const. stress.

* from point e, the strain hardening phenomenon becomes predominant and the strength of the material increases, thereby requires more stress for deformation, until point f is reached. point f is known as ultimate point and stress corresponding to this stress known as ultimate stress. σ_u.

it is the maximum stress to which material can be subjected in a simple tensile test.

* At point f the necking of the material begins and the cross-section area start decreasing at a rapid rate

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Due to this local necking, the stress in material goes on decreasing inspite that the actual stress intensity goes on Increasing. Ultimately the specimen breaks at point g, known as breaking point. corresponding stress know as breaking stress based upon the original cross section area.

Relation between elastic constant:-

(i) $E = 2G(1+\mu)$

(ii) $E = 3K(1-2\mu)$

(iii) $E = \dfrac{9KG}{3K+G}$

where E = Modulus of elasticity
G = Modulus of regidity
K = bulk modulus.
μ = poission ratio.

R.K.

Q: A concrete cyliender of diameter 150 m.m. and length 300 m.m. when ~~suj~~ subjected to an axial compressive load of 240 KN ~~res~~ resulted in an increase of diameter by ·127 m.m. and a decrease in length of ·28 m.m. Calculate the value of poission ratio and modulus of elastic. (E).

Sol:- Given that:-

$d = 150\,mm = \cdot 15\,m.$

$l = 300\,m.m = \cdot 3\,m.$

$P = 240\,KN = 240 \times 10^3\,N.$

$\delta d = +\cdot 127\,m.m. = \cdot 127 \times 10^{-3}\,m.$

$\delta l = -\cdot 28\,m.m = -\cdot 28 \times 10^{-3}\,m.$

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$\therefore$ linear strain $= \frac{\delta l}{l} = \frac{.28 \times 10^{-3}}{.3} = .000933$

lateral strain $= \frac{\delta d}{d} = \frac{.127 \times 10^{-3}}{.150} = .000846$

$\therefore$ Poission ratio :- $\frac{\text{lateral strain}}{\text{linear strain}}$

$= \frac{.000846}{.000933}$

$\mu = .907$ Ans

and $\delta l = \frac{Pl}{AE}$

$E = \frac{P \times l}{A \times \delta l}$ on solving

$E = 14.55 \times 10^{9}$ N/m². Ans

Q:- the following data related to a bar subjected to tensile test :-

d = 30 m.m., Tensile load P = 54 KN, Gauge length l = 300 m.m.
Extension of the bar δl = .122 m.m. Change in dia, δd = .00366 mm
find :- (i) poission ratio (ii) the value of three moduli.

Sol:-

μ = poission ratio $= \frac{\text{lateral strain}}{\text{linear strain}}$

$= \frac{\delta d / d}{\delta l / l}$

$= \frac{1.22 \times 10^{-4}}{\cancel{3.73 \times 10^{-4}}} = \cancel{.327}$ .300 Ans

4.066×10^{-4}

and: $E = \frac{\text{stress}}{\text{strain}}$

$$= \frac{54\times10^3 / \pi/4 \times .03^2}{3.73\times10^{-4}}$$

$$= 2.05\times10^5 \ MN/m^2. \text{ Ans}$$

$\therefore E = 2G(1+\mu)$

$$G = \frac{E}{2(1+\mu)} = .77\times10^5 \ MN/m^2 \text{ Ans}$$

and

$E = 3K(1-2\mu)$

$$K = \frac{E}{3(1-2\mu)}$$

$$= 1.97\times10^5 \ MN/m^2 \text{ Ans}$$

Thermal stress & strain:-

—x—x—x—

Stress which induced in a body due to change in the temperature is known as thermal stress and the corresponding strain known as thermal strain

let l = length of a bar of uniform cross-sectional area
t_1 = initial temperature of bar
t_2 = final temperature.
α = coefficient of linear expansion.

So extension of the bar due to rise in temperature

$$\delta l = \alpha \times (t_2 - t_1) \times l.$$

~~it thermo~~ if this elongation of the bar is prevented by some external forces. temperature strain will be given by:-

thermal strain $= \frac{\delta l}{l} = \frac{\alpha(t_2 - t_1) \times l}{l}$

$$\boxed{\epsilon_T = \alpha(t_2 - t_1).}$$

and

thermal stress developed $= \alpha(t_2 - t_1) \times E$

* when body is free to expand than thermal stress will be zero.
* when temperature increases then thermal stress nature will be compressive
* when temperature decreases then thermal stress nature will be tensile.

Strain Energy:- When an elastic body is loaded it undergoes deformation i.e. it's dimensions change. and when it is relieved of the load it regains it's orignal shape. for the time of loading energy is stored in it, the same is released when load is removed. This energy is called strain Energy.

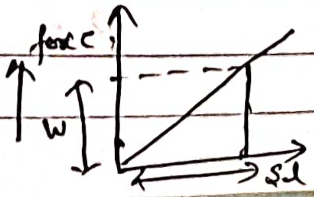
The strain energy stored by the body within elastic limit when loaded externally is known as "resilience" and the maximum which a body stores up to elastic limit is called "Proof resilience."

(1) Due to gradually applied load:-

Strain energy stored in the bar = Work done by the load

$U =$ Avg. load $\times$ distance.

$U = \frac{W}{2} \times \delta l$



Q:- A steel bar 4cm. by 4cm. in section, 3m. long is subjected to an axial pull of 128 kN. Taking $E = 200\ GN/m^2$ Calculate the alteration in the length of the bar. Calculate the amount of energy stored in the bar during extension.

Sol:- Given:-

Area $= 4cm \times 4cm = 16\ cm^2$

$= 16 \times 10^{-4}\ m^2$

$W = 128\ kN$ $E = 200 \times 10^9\ N/m^2$

$\delta l = ?$ energy stored during elongation i.e. strain Energy $U =$

$l = 3m.$

$$\delta l = \frac{W \times l}{A \times E}$$

$$= \frac{128 \times 10^3 \times 3}{16 \times 10^{-4} \times 200 \times 10^9} = .0012\ m.$$ Ans

Energy stored during Elongation i.e. $U = \frac{\sigma^2 \times A \times l}{2 \times E}$

$$U = \frac{(8 \times 10^7)^2 \times 16 \times 10^{-4} \times 3}{2 \times 200 \times 10^9}$$ $\because \sigma = \frac{W}{A}$

$U = 76.8\ N\text{-}m$

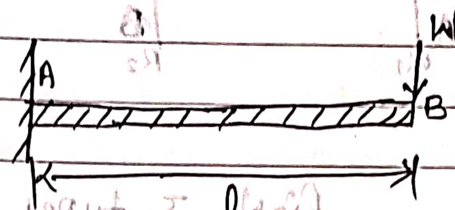
$U = 76.8\ J$ Ans

Unit - II
Shear forces and bending Moment

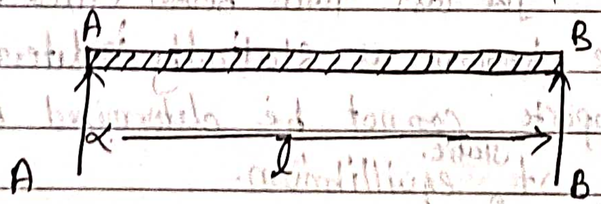
Beam :- the beam is defined as the structural member which is used to bear different loads. It resists Vertical load, shear forces & bending moments.

Types of beam :-

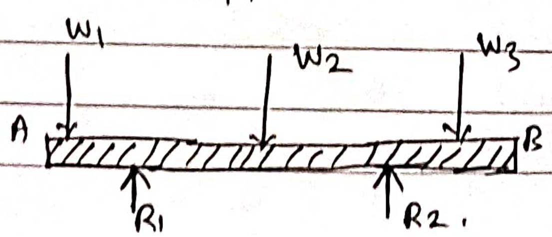
(i) **Contilever beam :-** A Contilever beam is a beam ~~whereas~~ whose one end is fixed and other end is free.



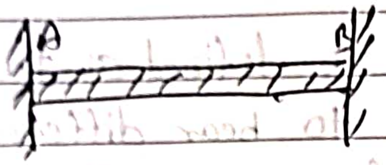
(ii) **Simply-supported beam :-** A beam which is supported or resting on the support at it's both the ends, is called simply supported beam.



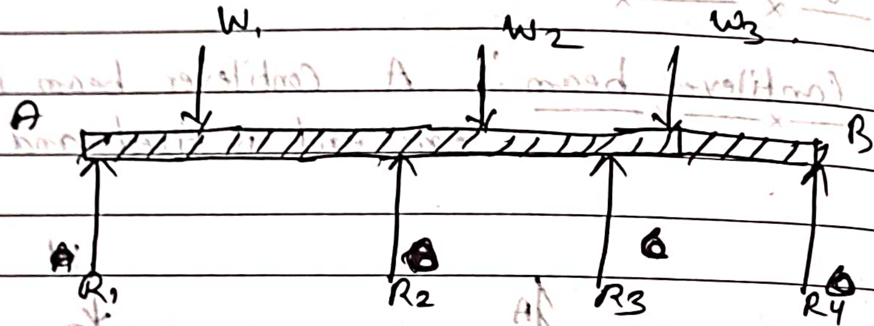
(iii) **Overhanging beam :-** An over hanging beam is one in which the supports are not situated at the ends.



(iv) Fixed beam :- A beam which has both of it's ~~beam~~ end fixed or built in walls is called fixed beam.



(V) Continuous beam :- A beam which has more than two supports.



First 3 types of beam (cantilever, Simply supported, Overhanging beams) are ~~subjected~~ known as statically determinate beam. As the reaction of these beam at their support can be determine by the uses of ~~static equilibrium~~ equation of static equillibrium.

the last two beam (fixed beam & continuous beam) are known as statically indeterminate beam as their supports cannot be determined by the use of equation of static equillibrium.

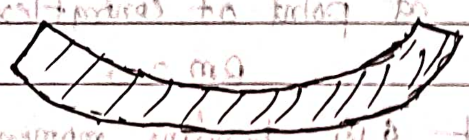
Shearing force:- It is defined as the algebric sum of forces acting either on left hand side or right hand side of section.

Unit - N. (Newton)

Bending Moment:- the moment produced by the forces acting on the beam must be ~~compom~~ balanced by equal and opposite moment produced by ~~the~~ ~~are~~ the internal forces acting on the beam at the section. this is the bending moment at that section.

Unit - N-m.

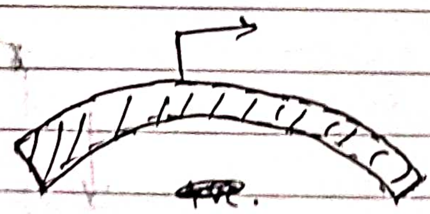
the bending moment which causes a beam to bend with the ~~a~~ concave side upwards, is called a sagging bending moment, and it is taken as positive.



Sagging (+ve)

* in sagging type. the upper part is ~~in~~ under compression stress and lower part is ~~tensilee~~ under tensile stress.

the bending moment causing convex-ity upwards will be taken as negative and called Hogging bending moment.



Hogging (-ve)

* in Hogging type the upper part is under tensile ~~to~~ stress and lower part is under compressive stress.

Relation between shear force and bending moment -

$$w = \frac{dF}{dx}$$ where

w = intensity of load

F = shear force.

and

$$F = \frac{dM}{dx}$$ where

F = shear stress

M = Bending moment.

Imp

Point of contraflexure :- In a beam if bending moment changes sign at a point, the point itself having zero bending moment, the changes curvature at this point of zero bending moment and this point is called the point of contraflexure or point of inflexion or point of virtual hinge.

So at point of contraflexure.

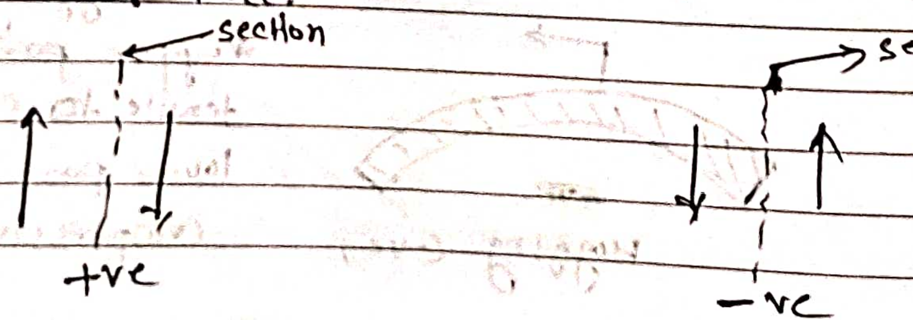
$BM = 0$

* It is a point in bending moment daigram in which Bending moment changes it's sign from +ve to -ve or -ve to +ve

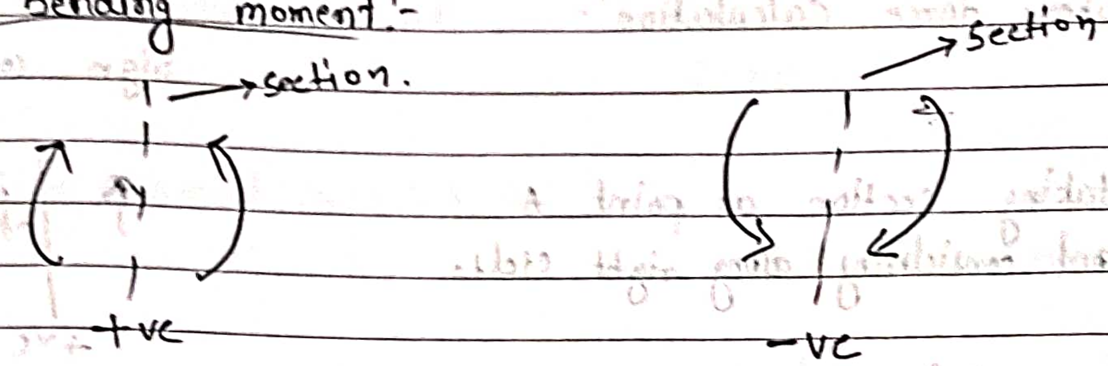
* Point of contraflexure is that poin in the beam which is not bending..

Sign Convention :-

For shear force :-



for bending moment :-

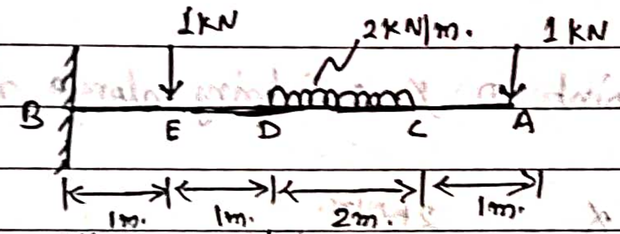


* this is the generally used sign convention. Some books may use different sign convention.

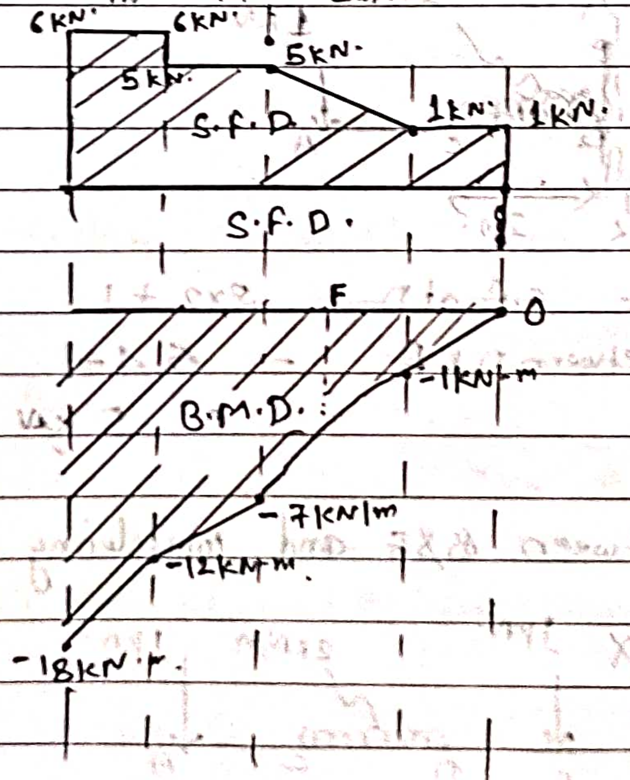
Numericals :-

(A) Cantilever beam :-

Q:-1 Draw the shear force & Bending moment diagrams for cantilever beam loaded as shown in fig.

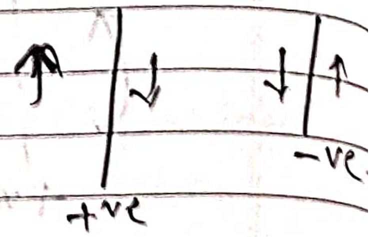


Sol:-



Shear force calculation:-

Sign convention

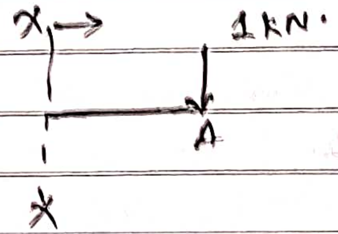


taking section at point A and considering along right side.

S.f. = 0 kN.

taking section between point C & A and considering right side.

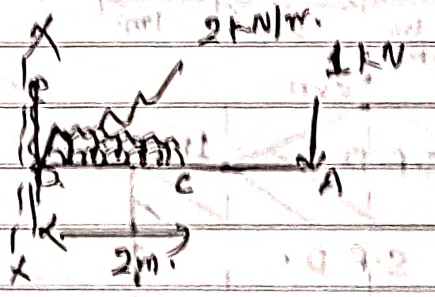
F.B.D



$\therefore$ S.f.$_{AC}$ = 1 kN.

taking section at point D & considering along right hand side.

F.B.D:-

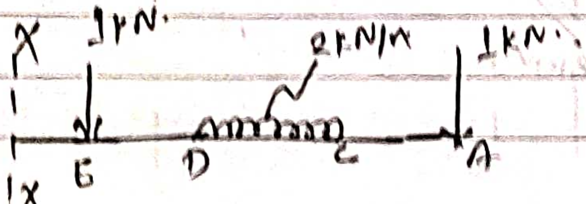


~~S.f.D =~~ ~~S.f at D =~~ 2x2 + 1

S.f between D & G = ~~5~~ 4+1

= 5 kN

taking section between B & E and considering along right hand side -

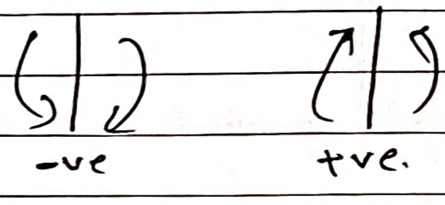


S.F. between B & E $= 1 + 2\times2 + 1$

$= 6 \text{ kN}.$

Bending moment calculation -

Sign convention.



~~B.M.~~ At point A :-

taking section at point A and considering along R.H.S.

$$(B.M.)_A = 0$$

At point C :-

taking section at point C and considering along R.H.S. :

$$(B.M.)_C = -1 \times CA$$
$$= -1 \times 1$$
$$(B.M.)_C = -1 \text{ kN-m}.$$

taking Section at point D and Considering along R.H.S.

$$B.M._D = -2\times2\times1 - 1\times3.$$
$$= -4-3$$
$$= -7 \text{ kN-m}$$

taking section at point E and considering along R.H.S.

$$(B.M.)_E = -2\times2\times\left(1+\frac{2}{2}\right) - 1\times4$$

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$= -8 - 4 = -12$ kN-m.

taking section at B and considering R.H.S.:-

$$(B.M.)_B = -1\times1 - 2\times2\left(1+1+\frac{2}{2}\right) - 1\times5$$

$$= -1 - 12 - 5$$

$$= -18 \text{ kN-m.}$$

∴ portion betn C & D is loaded with U.D.L. so the Bending moment diagram will be parabolic curve so we have to find a point in between to draw the curve.

So letn consider a point F at the middle of C & D ~~at~~ and

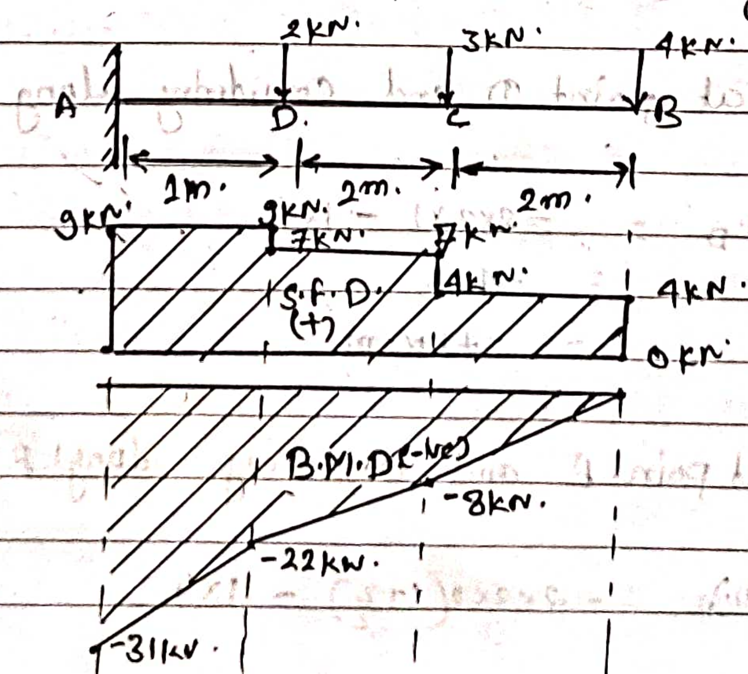
$$(B.M.)_F = -\cancel{2}\times1\times\frac{1}{\cancel{2}} - 1\times2$$

$$= -1 - 2$$

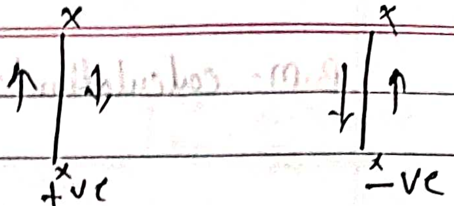
$$= -3 \text{ kN.m.}$$

Q:- 2 Draw the S.F.D & B.M.D. of given beam:-

Sol:-



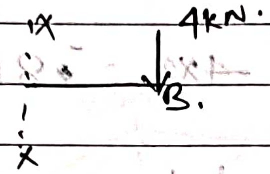
Shear force calculation:-



considering section at B & considering along right side.

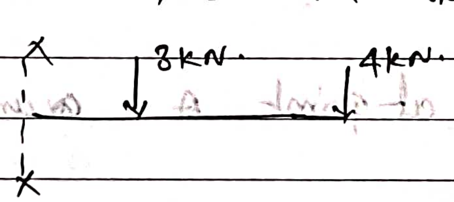
$$S.F._{at\ B} = 0\ KN.$$

~~considering~~ taking section ~~at~~ between B & C and considering along R.H.S.



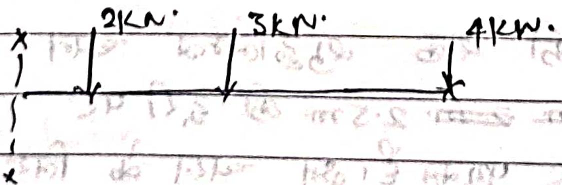
$$S.F.\ between\ B \& C = +4KN.$$

taking section, between C & D and considering along R.H.S.



$$S.F._{x\text{-}x}\ between\ C \& D = 4+3$$
$$= 7KN.$$

taking section between A & D and considering along R.H.S. =



$$S.F.\ bet^n_{A \& D} = +2+3+4$$
$$= 9KN.$$

B.M. calculation:-

$-ve$ $+ve$

taking section at B & considering along R.H.S:-

$$B.M._B = 0$$

taking section at point C & considering along R.H.S.

$$B.M._C = -4\times2 = -8\ kN\text{-}m.$$

taking section at point D and considering along R.H.S.

$$B.M._D = -3\times2 - 4\times4$$
$$= -22\ kN.m.$$

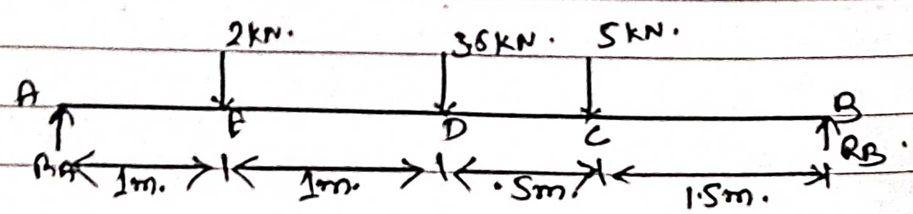
taking section at point A and considering along R.H.S.

$$B.M._A = -2\times1 - 3\times3 - 4\times5$$
$$= -31\ kN.m$$

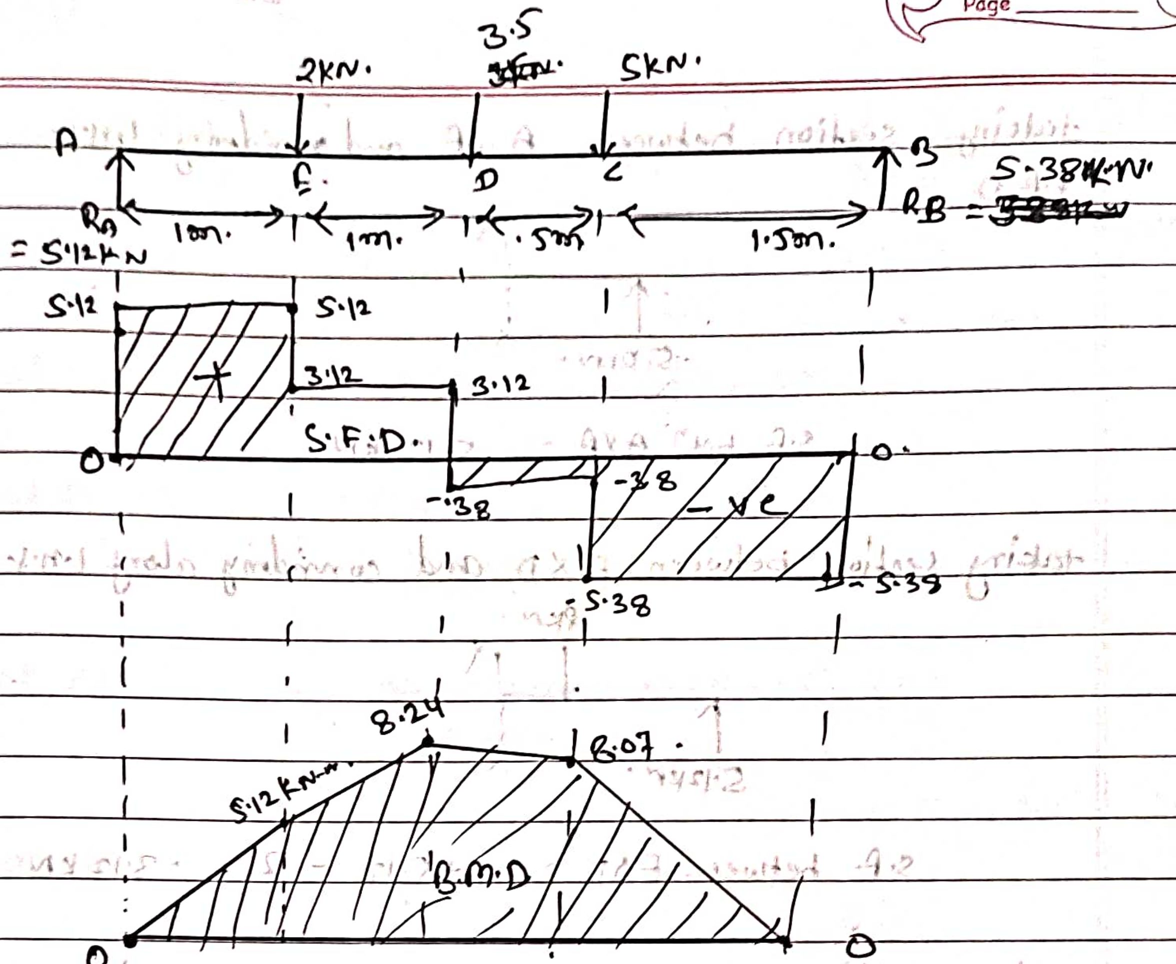
Ans

B. Simply supported Beam:-

Q. 1 4 m. लम्बाई की एक शुद्धालम्ब धरन के बांये आलम्ब से 1m, 2m. और ~~2m~~ 2.5m. की दूरी पर क्रमशः 2 kN, 3.5 kN, और 5 kN. के भार प्रयुक्त है। इस धरन के लिए अपरूपण बल आरेख एवं बंकन आघूर्ण आरेख बनाओ।



Sol:-



$$\Sigma F_y = 0$$

$$R_A + R_B = 2 + 3.5 + 5$$

$$R_A + R_B = 10.5\,kN \quad \text{---} \; (1)$$

$$\Sigma M_A = 0$$

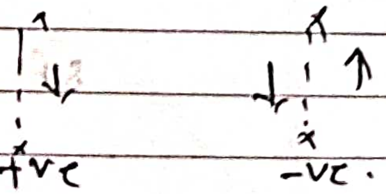
$$R_B \times 4 = 2\times 1 + 3.5\times 2 + 5\times 2.5$$

$$R_B = 5.38\ kN.$$

$$R_A = 5.12\ kN.$$

Shear force calculation:-

Sign convention

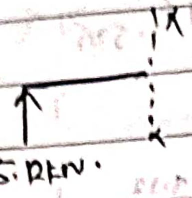


taking section at A and considering L.H.S.

$S.F_A = 0.$

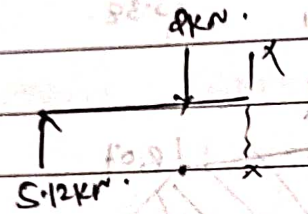
taking section between A & E and considering L.H.S:

F.B.D.



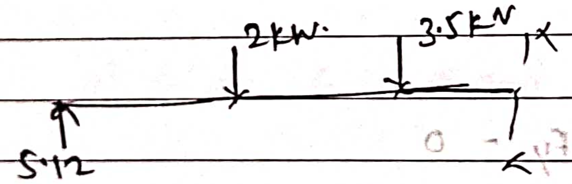
S.F bet^n A & E = 5.12 kN.

taking section between E & D and considering along L.H.S. -



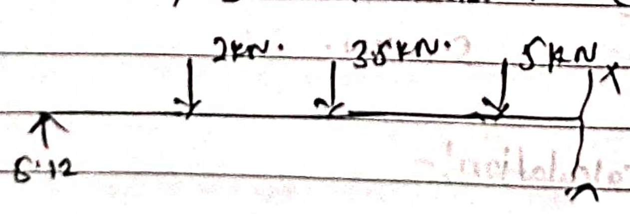
S.F. between E&D = +5.12 - 2 = 3.12 kN.

taking section between D & C and consideri L.H.S.



S.F. bet^n D&C = +5.12 - 2 - 3.5
= -.38 kN.

taking section bet^n. C & B and considerin. L.H.S.

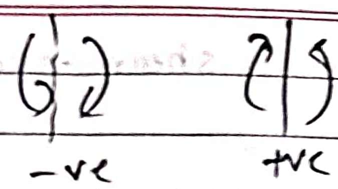


S.F. bet^n C&B = 5.12 - 2 - 3.5 - 5
= -5.38 kN.

taking section at B and considering R.H.S -

S.F.$_B$ = 0.

B.M. calculation.



B.M. at A = 0.

B.M at E = +5.12×1 = 5.12 kN-m.

B.M. at D = +5.12×2 − 2

= 8.24 kN.m.

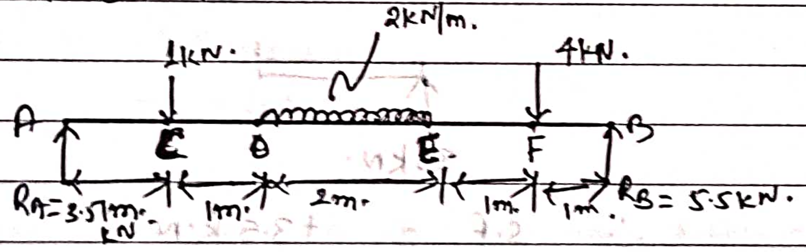
B.M. at C = ~~+5.12×2.5~~ 5.38×1.5

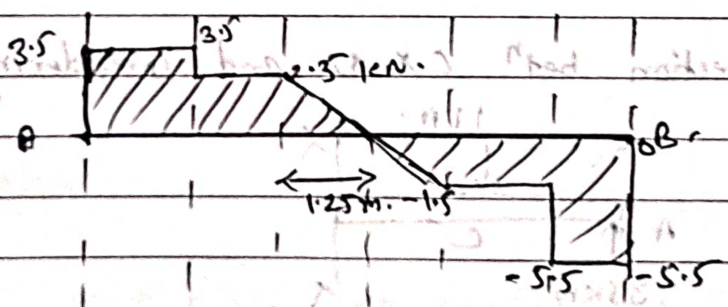
= +8.07 kN.m.

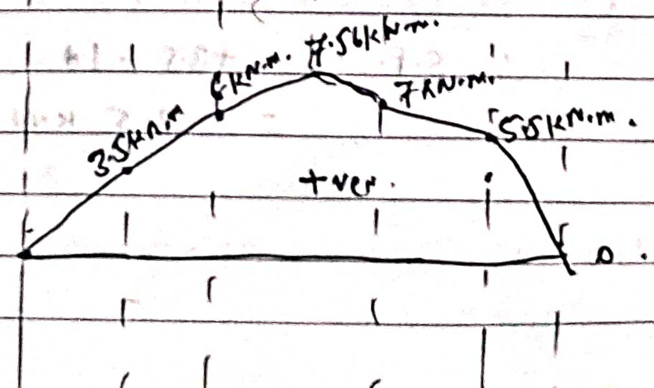
B.m at B = 0.

Q.2 Draw the S.F. & B.M. diagram for the beam shown in fig. & find the max. B.M.

Sol:-







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B.M.C :- taking section at C and considering L.H.S.

$$B.M._C = 3.5 \times 1 = 3.5 KN\text{-}m.$$

taking section at D and considering L.H.S :-

$$B.M._D = 3.5 \times 2 - 1 \times 1$$
$$= 6 KN\text{-}m.$$

taking section at E and considering L.H.S. :-

$$B.M._E = 3.5 \times 4 - 1 \times 3 - 2 \times 2 \times 1$$
$$= 7 KN\text{-}m.$$

taking section at F and considering R.H.S :-

$$B.M._F = 5.5 \times 1$$
$$= 5.5 KN\text{-}m.$$

Max. B.M. occur at the point where S.F. = 0.

So :-

S.F. betn D and E will be given by the eqn

$$= 2.5 - 2x$$

$$\therefore 2.5 - 2x = 0$$
$$2x = 2.5$$
$$x = 1.25 m.$$

$\therefore$ Max. B.M. will be produce at 1+1+1.25 m.
3.25 m. from point A.

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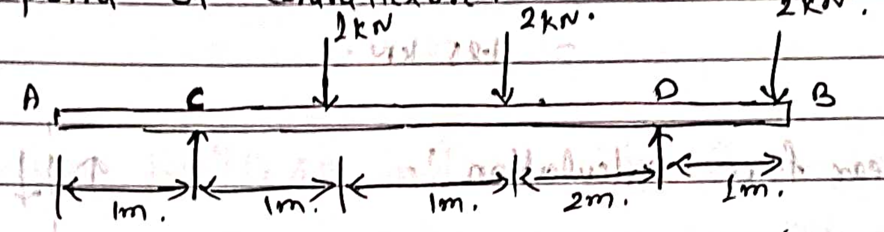
$\therefore$ B.M.max $= 3.5 \times 3.25 - 1 \times 2.25 - 2 \times 1.25 \times \frac{1.25}{2}$.

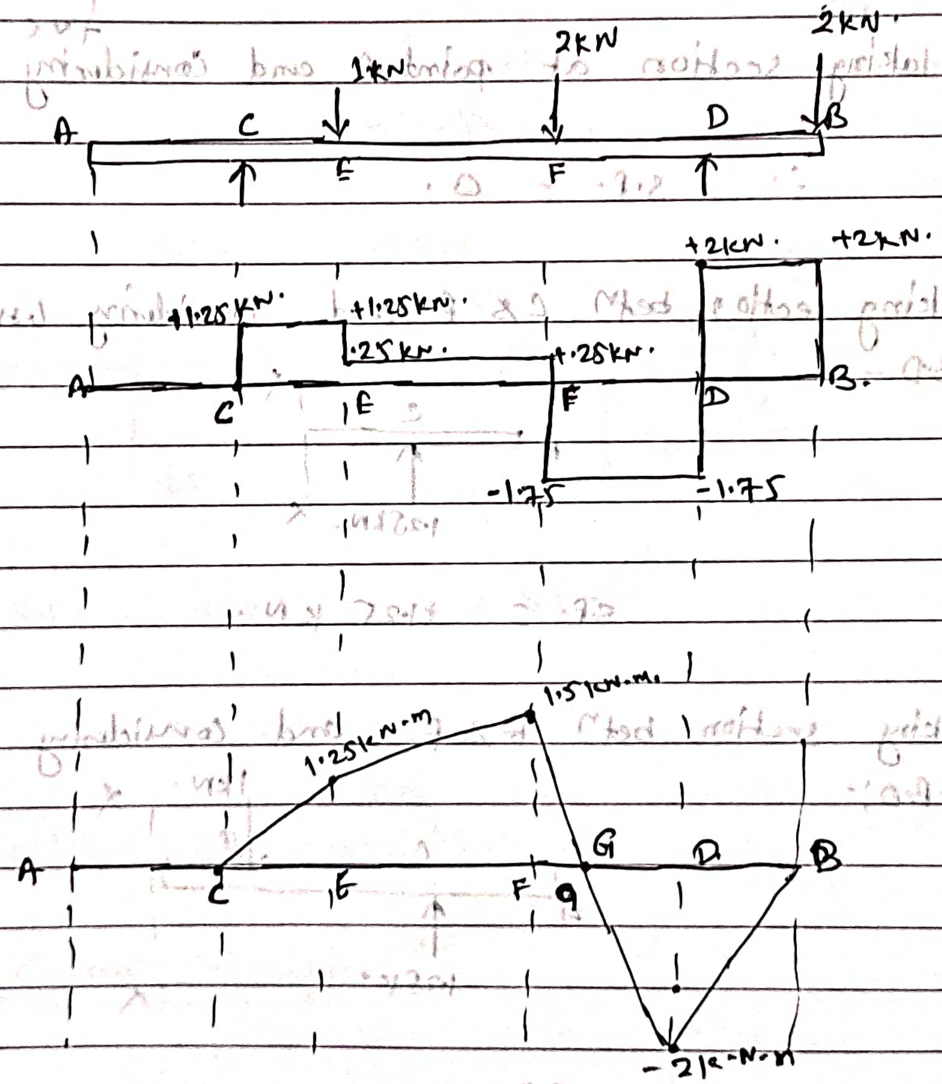
$= 7.56$ kN-m. Ans

C Over hanging beam :-

Q:-1 Draw the S.F. & B.M. diagram for given beam and find the point of contraflexure.



Sol:-



$R_C + R_D = 1+2+2 = 5$ ——— ①

$\Sigma M_C = 0$

$R_D \times 4 = 1\times1 + 2\times2 + 2\times5$

$= 1+4+10$

$R_D = \frac{15}{4} = 3.75$ KN. Put the value in eq. ①

$\therefore R_C = 5 - 3.75$

$= 1.25$ kN.

Shear force calculation :-

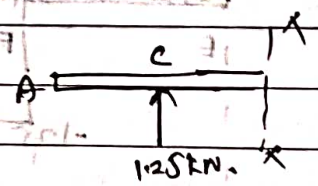
+ve   -ve.

taking section at point A and considering L.H.S.

$\therefore$ S.f. = 0.

taking section betn C & E and considering L.H.S.

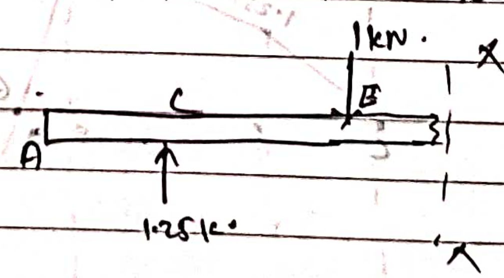
f.B.D -



S.f. = +1.25 kN.

taking section betn E & F and considering L.H.S.

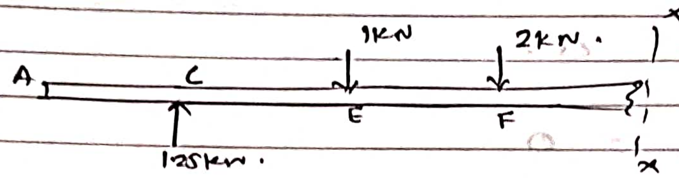
f.B.D:-



S.f. = +1.25 - 1.

= .25 kN.

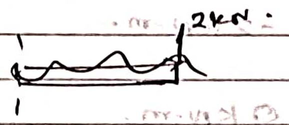
taking section betn F & D and considering L.H.S. :-
F.B.D :-



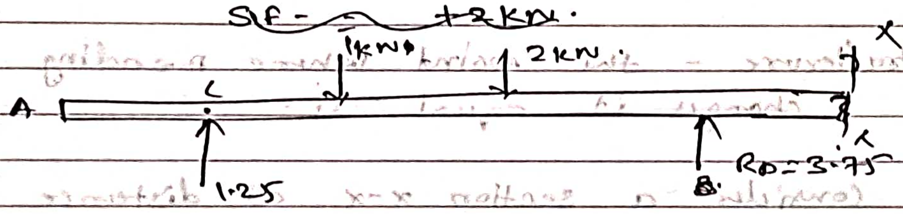
S.F = +1.25 − 1 − 2
= −1.75 KN.

taking section betn D & B and considering. ~~R.H.S.~~ L.H.S.

F.B.D :-



~~S.F = +2KN.~~

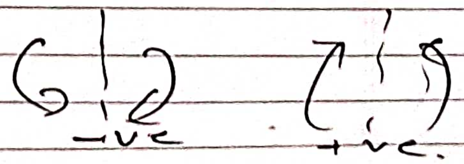


S.F = +1.25 − 1 − 2 + 3.75

= 2 KN.

~~taking section at B considering R.~~

Bending moment calculation :-



taking section at A and considering L.H.S.

$BM_A = 0$.

and $B.M._C = 0$

$B.M._E = +1.25 \times 1 = 1.25$ kN.m.

$B.M_F = +1.25 \times 2 - 1.\cancel{25} \times 1$

$= +1.5$ kN.

$B.M._D = -2 \times 1$

$= -2$ kN.m.

$B.M._B = 0$ kN.m.

point contraflexure - the point where Bending moment ~~changes it~~ equal $= 0$.

let's consider a section x-x at distance x from point B

$\therefore$

$BM_x = +3.75(x-1) - 2 \times x$

$\therefore$ at point of contraflexure. $BM. = 0$

$\cancel{3.75}\ 3.75x - 3.75 - 2x = 0$

$1.75x = 3.75$

$x = \frac{3.75}{1.75}$

$x = 2.143$ m.

Ans

So point of contraflexure is situated at a distance 2.145m. from point B.

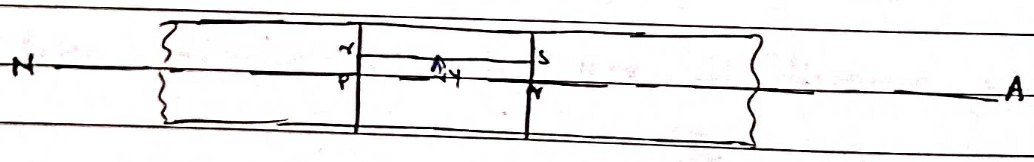
Unit - III

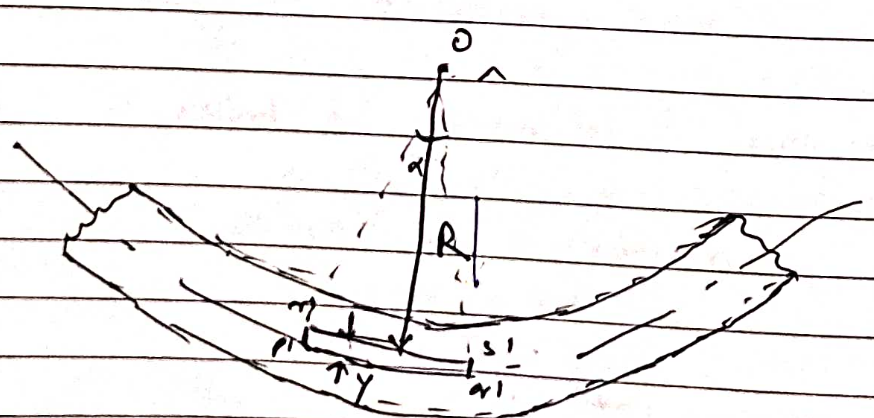
Bending stresses and shear stresses in beam type components.

Theory of simple bending :-

Assumption :-

1. The material of the beam is perfectly homogeneous throughout
2. the stress is directly proportional to strain and at no place stress exceed the elastic limit.
3. the value of modulus of elasticity (E) is same, for the fibr of the beam under compression or ~~wal~~ under tension.
4. There is no resultant pull or push on the cross-section of the beam.
5. the radius of curvature of the beam before bending is very ~~laad~~ large in comparison to its tranverse dimensions.





$p'q' = R\alpha$

$r's' = (R-y)\alpha$

Initially $pq = rs$

and $pq = p'q'$ $\because$ (No strain in N.A.)

$\therefore$ strain in $rs = \dfrac{rs - r's'}{rs}$ but $rs = pq = p'q'$

$= \dfrac{p'q' - r's'}{rs} = \dfrac{R\alpha - (R-y)\alpha}{R\alpha}$

$= \dfrac{\cancel{R\alpha} - \cancel{R\alpha} + y\alpha}{R\alpha} = \dfrac{y}{R}$

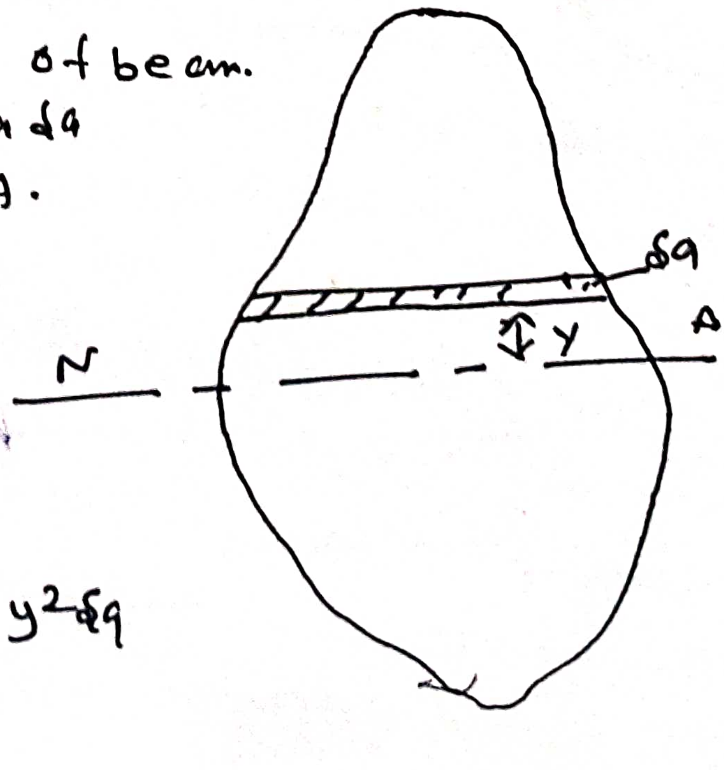
if stress in rs is σ and modulus of elasticity $= G$

then strain in rs : $\dfrac{\sigma}{E}$

$$\therefore \frac{\sigma}{E} = \frac{Y}{R}$$

$$\frac{\sigma}{Y} = \frac{E}{R} \quad \text{------ (i)}$$

$\therefore$ Consider a tranverse section of beam.
and consider a strip of Area δa
at a distance y from N.A.



$\therefore$ Normal force on the Area $= \dfrac{E}{R} y\, \delta a$

Resisting moment $= \dfrac{E}{R} y^2 \delta a$

and the total resisting moment is

$$= \Sigma \frac{E}{R} y^2 a \, \delta a$$

$$= \frac{E}{R} \Sigma y^2 \delta a$$

$\Sigma y^2 \delta a$ is the second moment of area about Neutral Axis

$\therefore$ Total Resisting moment $= M = \frac{E}{R} \times I$

$$\therefore \frac{M}{I} = \frac{E}{R}$$

from eq. 5.1

$$\boxed{\frac{M}{I} = \frac{\sigma}{Y} = \frac{E}{R}}$$

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final equation is -

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Where:-
M = moment of resistance
I = Moment of inertia of the section about Neutral Axis (N.A.)
E = Young's Modulus of Elasticity
R = Radius of curvature of N.A.
σ = bending stress.

Imp
* Neutral Axis or Neutral layer passes through the centre of area.

Imp
Section Modulus :-

from equation $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma = \frac{M}{I/y}$$

$$\sigma = \frac{M}{Z} \qquad Z = \frac{I}{y}$$

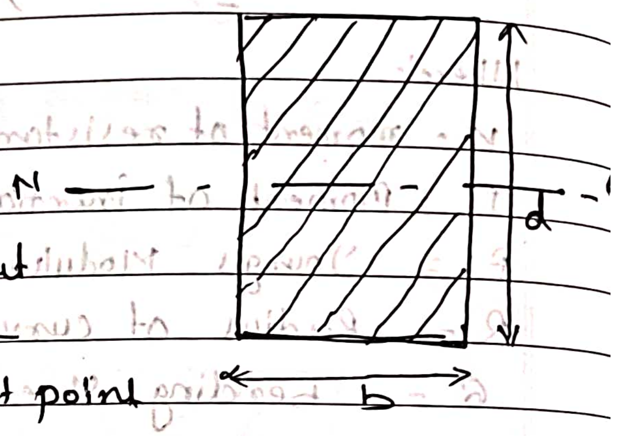
Z = section modulus.

The strength of the beam depends mainly on the section modulus.

Unit - m^3.

The section modulus for rectangular and circular sections are -

(i) Rectangular section :-



Section modulus $(Z) = \dfrac{\text{moment of inertia about neutral axis}}{\text{distance of the most distant point from neutral axis}}$

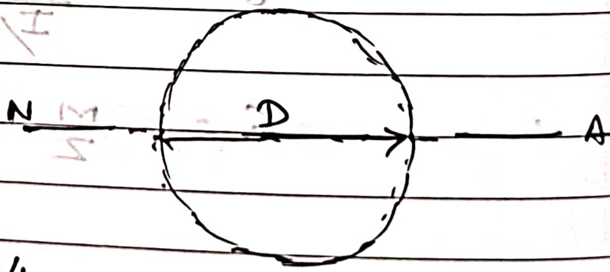
$$Z = \frac{I}{Y_{max}} \qquad I = \frac{bd^3}{12}$$

$$\therefore Y_{max} = d/2$$

$$Z = \frac{b \times d^{\cancel{3}\,2}}{\cancel{12}\,6} \times \frac{\cancel{2}}{\cancel{d}}$$

$$\boxed{Z = \frac{bd^2}{6}}$$

(ii) circular section :-



$$Z = \frac{I}{Y_{max}}$$

$$I = \frac{\pi}{64} \times D^4$$

$$Y_{max} = \frac{D}{2}$$

$$Z = \frac{\pi}{\cancel{64}\,32} \times D^{\cancel{4}\,3} \times \frac{\cancel{2}}{\cancel{D}}$$

$$\therefore \boxed{Z = \frac{\pi}{32} \times D^3}$$

Q: 1 A 250 m.m. (depth) x 150 mm. (width) rectangular beam subject to maximum bending moment of 750 kN-m. Determine -

(i) the max. stress in the beam

(ii) If the value of E for the beam material is 200 GN/m2, find out the radius of curvature for that portion of the beam where the bending is maximum.

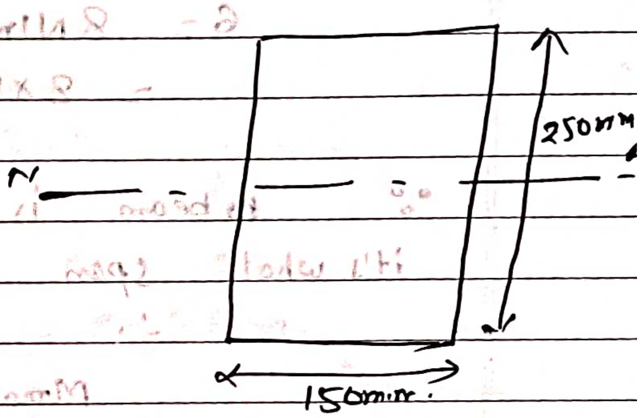
~~(iii) the value of longitudinal stress~~

Sol:- $d = 250$ m.m.

$b = 150$ m.m.

$M = 750$ kN-m.

$= 750 \times 10^3$ N-m.



$$I = \frac{b \times d^3}{12}$$

$$= \frac{.15 \times .25^3}{12} = .0001953\ m^4$$

$$y = d/2 = \frac{250}{2} = 125\ mm. = .125\ m.$$

$$\therefore \frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma = \frac{M}{I} \times y$$

$$= \frac{750 \times 10^3 \times .125}{.0001953} = 4.8 \times 10^8\ N/m^2$$

Ans

and $\frac{M}{I} = \frac{E}{R}$

$$R = \frac{E \times I}{M} = \frac{200 \times 10^9 \times .0001953}{750 \times 10^3}$$

$$R = 52.08\ m.$$ Ans

Q. 4m. विस्तृति (Span) की एक आयताकार परिच्छेदी शुद्धालम्ब धरन पर 20kN/m. का समवितरित भार (Udl) पूरी विस्तृति पर प्रयुक्त है। यदि धरन की गहराई और चौड़ाई का अनुपात 1.5 हो तो अनुमत प्रतिबल 8N/mm² के लिए गहराई और चौड़ाई का मान ज्ञात करो।

Sol: $l = 4m.$ $\quad w = 20 kN/m.$ $\quad d/b = 1.5$

$$\sigma = 8 N/mm^2$$
$$= 8 \times 10^6 N/m^2.$$

∵ beam is simply supported and have Udl on it's whole span

∴

$$M_{max} = \frac{wl^2}{8}$$
$$= \frac{20 \times 4^2}{8}$$
$$= 40 \ kN\text{-}m.$$

from bending equation:

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\frac{40 \times 10^3}{I} = \frac{8 \times 10^6}{y}$$

$$\therefore \frac{I}{y} = \frac{40 \times 10^3}{8 \times 10^6}$$

$$\frac{b \times d^{\cancel{3}\,2}}{\cancel{12}\,6} \times \frac{\cancel{2}}{\cancel{d}} = \frac{40 \times 10^3}{8 \times 10^6}$$

$$\frac{b \times d^2}{6} = \frac{40\times10^3}{8\times10^6}$$

$\because$ $d = 1.5 \times b$. put this value in above equation.

$$\frac{b \times (1.5 \times b)^2}{6} = \frac{40\times10^3}{8\times10^6}$$

$$b^3 = \frac{40\times10^3 \times 6}{1.5^2 \times 8 \times 10^6}$$

$\therefore$ $b = .237\ m.$ $\approx .24\ m.$

$\therefore d = 1.5 \times .24 = 0.36\ m.$

Answer

Q.3 A symmetrical section 200 m.m. deep has a moment of inertia of $2.26\times10^{-5}\ m^4$ about it's neutral axis. Determine the longest span over which, when simply supported, the beam would carry a uniformly distributed load of 4 kN/m run without the stress due to bending exceeding $125\ MN/m^2$.

Sol: $d = 200\ m.m.$

$I = 2.26 \times 10^{-5}\ m^4$

$l = ?$ $\quad$ $UDl = 4\ kN/m$

$\sigma = 125\ MN/m^2$.

Incase of ~~UDl~~ UDl in simply supported beam the max. bending moment is given by.

$$M = \frac{wl^2}{8}$$

$$M = \frac{4 \times l^2}{8}$$

from bending equation

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$M = \frac{\sigma}{y} \times I$$

$$= \frac{125\times10^6 \times 2.26\times10^{-5}}{.2/2}$$

$$M = 28.25\times10^3 \text{ N.m}.$$

and

$$M = \frac{wl^2}{8}$$

$$\therefore 28.25\times10^{3} = \frac{4\times10^{3}\times l^2}{8}$$

$$l^2 = \frac{28.25\times8^2}{4}$$

$$l = 7.516 \text{ m.}$$ Ans

Q:4 ~~A rectangular~~ 40m.m. व्यास के एक ठोस वृत्ताकार कैंटीउत्तोलक धरन की लम्बाई 2m. है। उसके मुक्त सिरे पर लगे अधिकतम बिन्दु भार का मान ज्ञात करो जिससे कि उत्पन्न बंकन प्रतिबल 50N/mm2 से अधिक न हो।

Sol:- $l = 2m.$ Beam - Cantilever.

$\sigma = 50\times10^6 \text{ N/m}^2$

let us assume load at the free end is $= W$ N.

so $M_{max} = W\times l = 2W$ N-m.

from bending equation -

$$\frac{M}{I} = \frac{\sigma}{y}$$

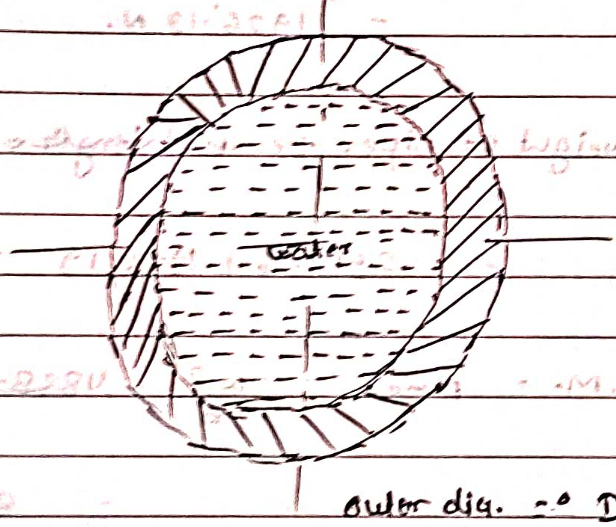
$$\frac{2\times W}{\frac{\pi}{64}\times .04^4} = \frac{50\times 10^6}{.02}$$

$$2\times W = \frac{50\times 10^6}{.02} \times \frac{\pi}{64} \times .04^4$$

$W = 157$ N. Ans

Q:- A cast-iron water main 12m. long of 500 m.m. inside dia. and 25m. wall thickness runs full of water and is supported at it's ends. Calculate the maximum stress in the metal if density of cast iron is 7200 kg/m³ and that of water is 1000 kg/m³.

Sol:-



Given:-

$l = 12$m.

$D_i = 500$mm $= .5$m.

$t = 25$m.m. $= .025$m

$\rho_{c.i.} = 7200$ kg/m3.

$\rho_w = 1000$ kg/m3.

Outer dia. $\therefore D_o = D_i + 2t$.

$= .5 + 2\times .025$

$= .55$ m.

Cross-section area of main $= \frac{\pi}{4}(D_o^2 - D_i^2)$

$= \frac{\pi}{4}(.55^2 - .5^2)$

$= .04123\ m^2$

weight of the water main in per meter length:-

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$$\rho = \frac{m}{V}$$

$$m = \rho \times V$$

$$m = \rho \times A \times l$$

$$W = mg.$$

$\therefore$

$$W = .04123 \times 1 \times 7200 \times 9.81$$

$$= 2912.6 \, N$$

Weight of water in one meter long main

$$W_w = \frac{\pi}{4} \times .5^2 \times 1 \times 1000 \times 9.81$$

$$= 1926.19 \, N.$$

Total weight of pipe per meter length

$$= 2912.16 + 1926.19 = 4838.35 \, N.$$

$\therefore$ Max. B.M. $= M_{max} = \frac{wl^2}{8} = \frac{4838.35 \times 12^2}{8}$

$$= 87090.3 \, Nm.$$

M.O.I. $= I = \frac{\pi}{64} \times [(.55)^4 - (.5)^4] = 1.42384 \times 10^{-3} \, m^4.$

$$y = \frac{D}{2} = \frac{.55}{2} = .275 \, m.$$

from bending eq^n. $\frac{M}{I} = \frac{\sigma}{y}$

$$\sigma = \frac{M \times y}{I}$$

$$= \frac{87090.3 \times .275}{142384 \times 10^{-3}} \times 10^{-6} = 16.82 \, MN/m^2$$

Ans

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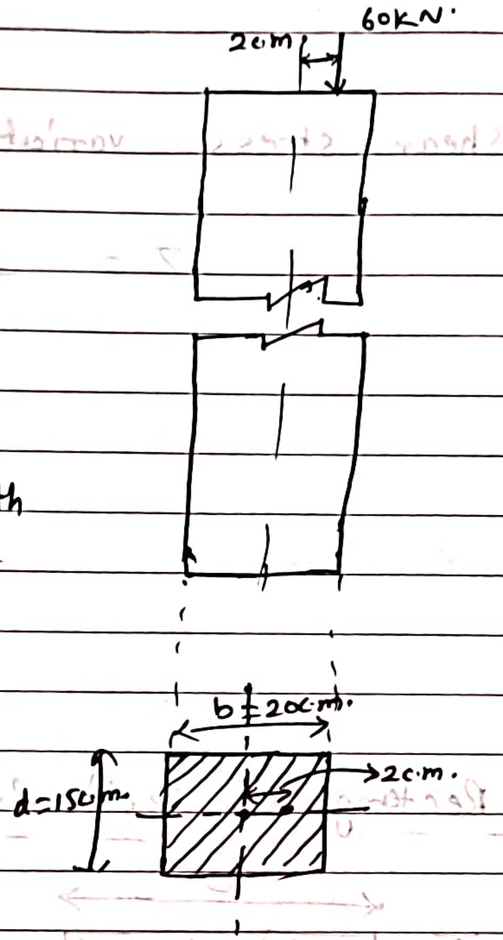
Q:- A rectangular strut is 20cm. wide and 15cm. thick. It carries a load of 60kN at an eccentricity of 2cm. in a plane bisecting the thickness. Find the max. and min. intensities of stress in the section.



Sol: Given:-
Width $= b = 20cm. = .2m.$
depth $= d = 15cm. = .15m.$
$e = 2cm. = .02m.$
$W = 60 kN.$

∴ load is not acting along the axis and have eccentricity of 2cm. so the will be 2 types of stress on work -

(i) direct stress
(ii) bending stress.

∴

direct stress $= \sigma_d = \dfrac{W}{A} = \dfrac{60}{.2 \times .15}$
$= .2 MN/m^2.$

bending stress $= \sigma_b = \dfrac{M}{Z}$

$= \dfrac{W \times e}{Z} = \dfrac{60 \times .02}{\dfrac{bxd^3}{12} \times \dfrac{2}{d}}$

$= \dfrac{60 \times .02}{\dfrac{.15 \times .2^2}{6}} =$

$\therefore \sigma_b = 1.2 MN/m^2.$

$\therefore$ Max. stress = $\sigma_d + \sigma_b$ = 2 + 1.2 = 3.2 MN/m2

Min stress = $\sigma_d - \sigma_b$ = 2 - 1.2 = .8 MN/m2.

Shear stress variation :-

$$\tau = \frac{SA\bar{y}}{Ib}$$

where:-

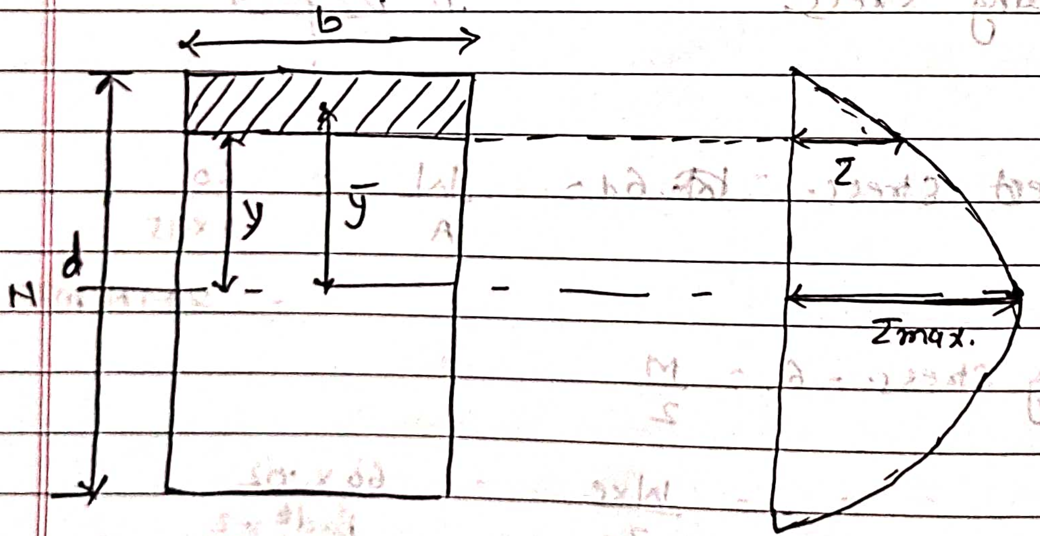
τ = shear stress, S = Shear force

A = Area of section where shear stress is to be calculated.

$\bar{y}$ = distance from N.A. to the centroid of the area.

I = M.O.I. of given fig. about N.A

(i) Rectangular section:-



let τ be the shear stress in a layer at a distance y from Neutral Axis.

$$\therefore \quad \tau = \frac{S \times A \times \bar{y}}{Ib}$$

A = Area of shaded portion

$$= b\left(\frac{d}{2} - y\right)$$

and $\bar{y} = \frac{\left(\frac{d}{2} - y\right)}{2} + y$

$$= \frac{d}{4} - \frac{y}{2} + y$$

$$= \frac{d}{4} + \frac{y}{2} = \frac{1}{2}\left(\frac{d}{2} + y\right)$$

$I = \frac{b \times d^3}{12}$ put value in formula.

$$\tau = \frac{S \times b\left(d/2 - y\right) \times \frac{1}{2}\left(\frac{d}{2} + y\right)}{\frac{b \times d^3}{12} \times b}$$

$$= \frac{12 \times S \times \cancel{b}}{\cancel{b \times d^3} \; \cancel{b} \times b d^3 \times 2} \times \left(\frac{d^2}{4} - y^2\right)$$

$$= \frac{6S}{bd^3} \times \left(\frac{d^2}{4} - y^2\right)$$

∴ at $y = 0$ $\tau = \tau_{max}$.

$$\therefore \tau_{max} = \frac{\overset{3}{\cancel{6}}S}{b \times d^{\cancel{3}}} \times \frac{d^{\cancel{2}}}{\cancel{4}_2} = \frac{3S}{2\,b \times d}$$

$\frac{S}{b \times d}$ = Mean shear stress (τ_{mean}).

$$\therefore \boxed{\tau_{max} = \frac{3}{2}\tau_{mean}}$$

(ii) Solid circular section:-

for circular section.

$$\tau = \frac{4}{3}\frac{S}{\pi R^4}(R^2-y^2)$$

at $y=0$ τ will be τ_{max}.

$$\therefore \tau_{max} = \frac{4}{3}\frac{S}{\pi R^{\cancel{4}2}} \times R^{\cancel{2}}$$

$$= \frac{4}{3}\frac{S}{\pi R^2}$$

$\frac{S}{\pi R^2}$ = will be taken as mean shear stress (τ_{mean}).

$$\therefore \boxed{\tau_{max} = \frac{4}{3}\tau_{mean}}$$

Q:- A circular beam 150 m.m. diameter is subjected to a shear ~~stress~~ force of 7 kN. calculate the value of max. shear stress. and sketch the variation of shear stress along the depth of the beam.

Sol:- Given that:-

$d = 150\,mm = .15\,m$

$S = 7\,kN = 7000\,N$

$$\tau_{mean} = \frac{S}{A} = \frac{S}{\frac{\pi}{4} \times d^2}$$

$$= \frac{7 \times 10^3}{\frac{\pi}{4} \times .15^2} = 396\ kN/m^2.$$

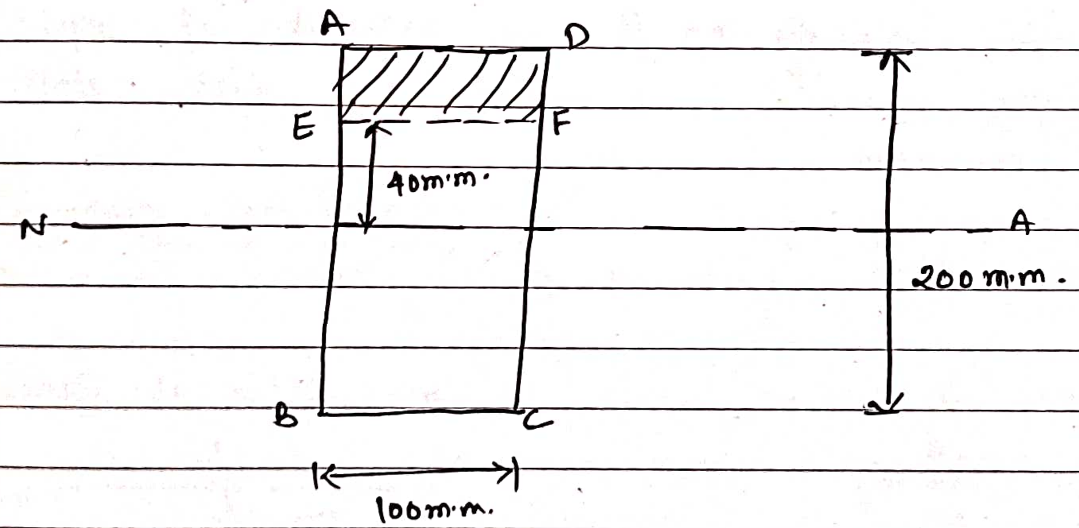
for Circular section

$$\tau_{max} = \frac{4}{3} \tau_{mean}.$$

$$\tau_{max} = \frac{4}{3} \times 396 = 528\ kN/m^2.$$ Ans.

Q:2 एक क्रांकीट की आयताकार धरन 100m.m. चौड़ी और 200m.m. गहरी है और दोनो सिरो में शुद्धालम्बित है। धरन के मध्यबिन्दु पर एक 20 KN का भार प्रयुक्त है। धरन की ~~उद~~ उदासीन अक्ष से 40m.m. की दूरी स्थित बिन्दु पर अपरूपण प्रतिबल ज्ञात करो।

Sol:-



$$\tau = \frac{S \times A \times \bar{y}}{I \times b}$$

$$\text{Max. Shear force} = \frac{W}{2} = \frac{20}{2} = 10\ kN.$$

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A = Area of ADEF $= 100 \times 60$

$= 6000$ mm² $= 6000 \times 10^{-6}$ m

and $\bar{y} = 40 + \frac{60}{2} = 70$ mm $= 70 \times 10^{-3}$ m

$I = \frac{b \times d^3}{12} = \frac{.1 \times .2^3}{12}$

put all the value in formula.

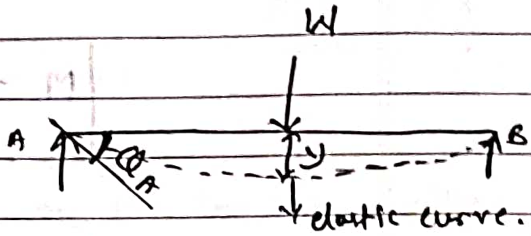
$$\tau = \frac{1 \times 10^3 \times 6000 \times 10^{-6} \times 70 \times 10^{-3}}{\frac{.1 \times .2^3}{12} \times .1}$$

$\tau = .63 \times 10^6$ N/m². **Ans**

Unit - 4

Deflection of beam type components.

Deflection :-



It is the vertical distance of the beam measured before and after loading.

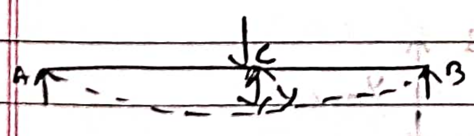
* It is denoted by y.
* Deflection at the supports is always zero.

Slope :- It is the angle in radian measured betⁿ the tangent to the elastic curve & the orignal of the beam.

* Slope is denoted by θ or $\frac{dy}{dx}$.
* It's unit is radians.

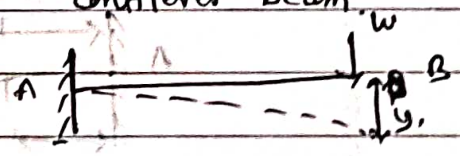
Boundary conditions :-

(i) Simply supported beam



* At A & B deflection = 0
* At C deflection = max.
* At C slope is = Zero.
* A A & B slope is = max.

(ii) Cantilever beam.



* At A deflection = 0
* At B deflection = max.
* At A slope = 0
* At B slope = max.

Relation betn slope, deflection & radius of curvature:-

$$\boxed{M = E \times I \times \frac{d^2y}{dx^2}}$$

where
M = Bending moment
I = M.O.I.
E = Modulus of Elasticity
y = ~~deflectd~~ deflection
$\frac{dy}{dx}$ = slope

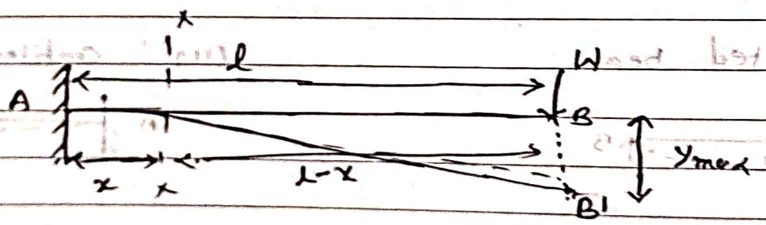
* the equation is based on Bending moment only. the effect of shear force is neglected.

The ~~imp~~ methods used for finding out the slope & deflection:-

(i) Double integration method
(ii) Moment Area method
(iii) Macaulay's Method.

Cantilever beam:-

I (imp) Cantilever beam with a point load W at freed end



A cantilever beam AB length l and fixed at point A, having a point load W at free end B.

let consider a section x-x at a distance x from fixed end. so Bending moment at section x-x.

$\dfrac{P|J}{+ve}$ $\dfrac{G|Z}{-ve}$

$$M_x = -W(l-x) \quad \text{----(i)}$$

$\because$ $EI\dfrac{d^2y}{dx^2} = M_x$ so eq (i) can be written as:

$$EI\dfrac{d^2y}{dx^2} = -W(l-x)$$

Integrating both side

$$EI\dfrac{dy}{dx} = -W\left(lx - \frac{x^2}{2}\right) + C_1 \quad \text{—(ii)}$$

$\because$ In case of cantilever beam at suppor slope $\dfrac{dy}{dx}$ will be equal to 0.

$\because$ at $x = 0$ $\dfrac{dy}{dx} = 0$. put the value in eq (ii)

$$C_1 = 0$$

$\therefore$

$$EI\dfrac{dy}{dx} = -W\left(lx - \frac{x^2}{2}\right) \quad \text{----(ii)}$$

$\therefore$ Slope at B = put $x = l$.

$$EI\dfrac{dy}{dx} = -W\left(l^2 - \frac{l^2}{2}\right)$$

$$= \frac{-Wl^2}{2}$$

$$\therefore \boxed{\frac{dy}{dx} = \theta_B = \frac{-Wl^2}{2EI}}$$

again integrating ~~eqn (iii) both st~~ eq. (iii) both side.

$$EIy = -W\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) + C_2$$

∴ in case of cantilever beam at $x = 0$ deflection $(y) = 0$

$$C_2 = 0$$ put the value in above eqn.

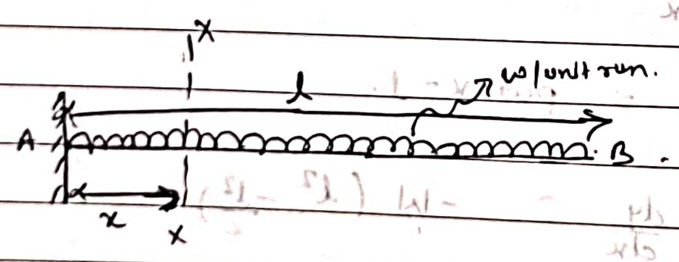
$$\therefore EIy = -W\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) + 0.$$

So deflection at point B will be put $x = l$,

$$EIy = -W\left(\frac{l^3}{2} - \frac{l^3}{6}\right)$$

$$\boxed{y_B = \frac{-Wl^3}{3EI}}$$ deflection at free end Ans.

II Cantilever beam ~~having~~ length l carrying udl w per unit length over whole length :-



Considering a cantilever beam with length l carrying a udl of w per unit meter run for whole length.

let take a section X-X ~~from~~ with distance x from fixed end. ~~and~~

So Bending moment along the section

x-x will be :-

$$M_x = -w(l-x) \times \frac{(l-x)}{2}$$

$$\therefore M_x = \frac{-w(l-x)^2}{2}$$

$\because$ $M_x = EI\frac{d^2y}{dx^2}$ So the above equation will be written as -

$$EI\frac{d^2y}{dx^2} = -\frac{w(l-x)^2}{2} \quad ---(i)$$

Integrating both side.

$$EI\frac{dy}{dx} = +\frac{w(l-x)^3}{6} + C_1$$

for ~~Cantilever~~ lever beam slope at fixed end will be zero. $\therefore$

at $x = 0$ $\frac{dy}{dx} = 0.$

$\therefore$ $\cancel{C_1 = 0}$

$$0 = \frac{w}{6}(l-0)^3 + C_1$$

$$C_1 = \frac{-wl^3}{6}$$ put this value in above equation and.

$$EI\frac{dy}{dx} = \frac{w(l-x)^3}{6} - \frac{wl^3}{6} \quad ---(ii)$$

So slope at free end will be put $x = l$.

$$\therefore EI\frac{dy}{dx} = 0 - \frac{wl^3}{6}$$

$$\left(\frac{dy}{dx}\right)_{at\ x=l} = \theta_B = \frac{-wl^3}{6EI}$$

again integrating equation (ii)

$$EIy = \frac{-w}{6\times4}(l-x)^4 - \frac{wl^3x}{6} + C_2$$

at $x = 0$ (fixed end) deflection $y = 0$.

$$\therefore 0 = -\frac{wl^4}{24} - 0 + C_2.$$

$$\therefore C_2 = \frac{wl^4}{24}$$ put the value in

above eq.

$$EIy = \frac{-w}{24}(l-x)^4 - \frac{wl^3x}{6} + \frac{wl^4}{24}$$

deflection at free end -

put $x = l$

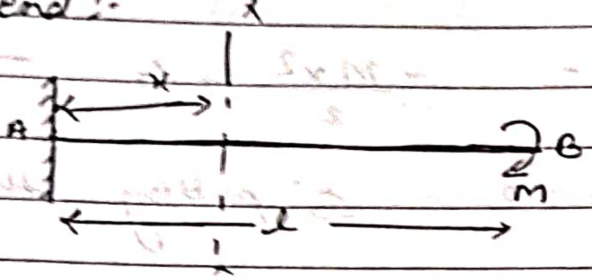
$$EIy = 0 - \frac{wl^4}{6} + \frac{wl^4}{24}$$

$$= \frac{-wl^4}{8}$$

$$\therefore \boxed{(y)_B = \frac{-wl^4}{8EI}}$$

deflection at free end B will be given by this equation.

III Cantilever of length l with moment applied at the free end:-



A cantilever beam AB of length l and fixed at point A and having a moment M at free end B.

So taking section x-x at a distance x from fixed end and taking moment along at x-x

$$M_x = -M$$

$\because$

$$M_x = EI \frac{d^2y}{dx^2}$$

$\therefore$

$$-M = EI \frac{d^2y}{dx^2}.$$

on integrating both side.

$$-Mx + C_1 = EI \frac{dy}{dx}. \quad \text{----} \ (1)$$

at $x = 0$ $\frac{dy}{dx} = 0$ putting the values in eq. (1).

$$\therefore \ 0 + C_1 = 0$$

$$C_1 = 0.$$

$$\therefore \ -Mx = EI \frac{dy}{dx}$$

again integrating both side.

$$-\frac{Mx^2}{2} + C_2 = EIy$$

$$EIy = -\frac{Mx^2}{2} + C_2 \quad \text{--- (1)}$$

at $x = 0$ $y = 0$ & putting this value in eq (1)

$\therefore$

$$0 = 0 + C_2$$
$$C_2 = 0.$$

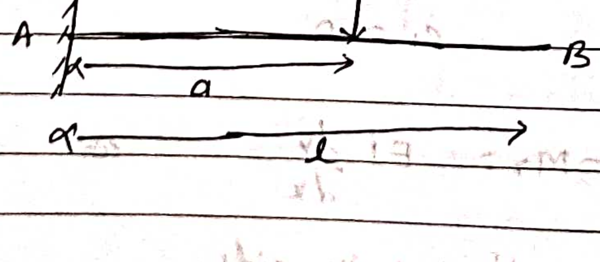
$$\therefore EIy = \frac{-Mx^2}{2}.$$

$\therefore$
$$\boxed{y = \frac{-Mx^2}{2EI}}$$

$\therefore$ deflection at free end $\therefore$ at $x = l$.

$$\boxed{y = \frac{-Ml^2}{2EI}}$$

Case iv Cantilever of length l carrying a point load W at distance a from the fixed end:-



$\therefore$ Slope:-

$$\theta_B = \theta_C = \frac{-Wa^2}{2EI}$$

and deflection will be given by:-

$$y_C = \frac{-Wa^3}{3EI}$$

deflection at free end:-

$$y_B = y_C + (l-a) \times \theta_B$$

$$= \frac{-Wa^3}{3EI} + (l-a) \times \frac{-Wa^2}{2EI}$$

$$\boxed{y_B = \frac{-Wa^3}{3EI} - (l-a)\frac{Wa^2}{2EI}}$$

Numericals:-

Q:1 100 m.m. चौड़ी और 200 m.m. गहरी एक कैंटी - उत्तोलक की लम्बाई 1.5 m. है। उसके आबद्ध सिरे से 1 m. की दूरी पर एक 10 KN का बिन्दु भार लगा है। मुक्त सिरे पर ढाल एवं विक्षेप ज्ञात करो।

$E = 2 \times 10^5 \ N/mm^2$

Sol:- Given:

$b = 100 m.m. = .1 m.$

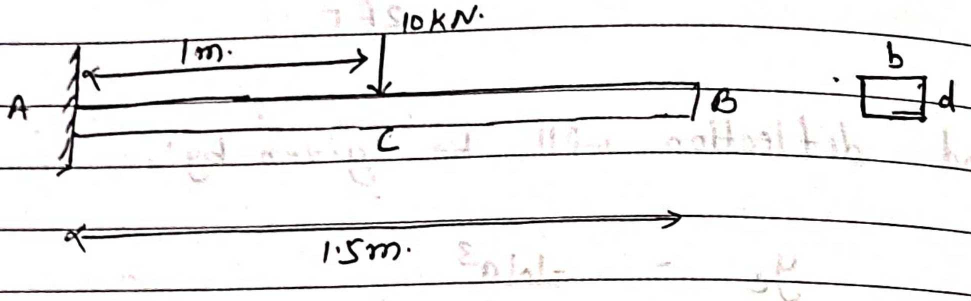
$d = 200 m.m. = .2 m.$

$l = 1.5 m.$

$a = 1 m.$ $W = 10 KN = 10 \times 10^3 N.$

$E = 2\times10^5\ N/mm^2 \quad = 2\times10^5\times10^6\ N/m^2$

$= 2\times10^{11}\ N/m^2.$



$\therefore \quad \theta_B = ?$

$y_B = ?$

$$\theta_B = \frac{-Wa^2}{2EI} \qquad I = \frac{b\times d^3}{12} = \frac{.1\times .2^3}{12}$$

$= 66.67\times10^{-6}\ m^4$

$$\theta_B = -\frac{10\times10^3\times1^2}{2\times2\times10^{11}\times66.67\times10^{-6}}$$

$= .00375$ radian.

and

$$y_B = y_c + (l-a)\ \theta_B$$

$$= -\frac{Wa^3}{3EI} - (l-a)\frac{Wa^2}{2EI}$$

$$= -\frac{10\times10^3\times1^3}{3\times2\times10^{11}\times66.67\times10^{-6}} - (1.5-1)\times\frac{10\times10^3\times1^2}{2\times2\times10^{11}\times66.67\times10^{-6}}$$

$y_c = -.4375$ m.m

$\hookrightarrow$ −ve sign shows the downward deflection.

$\therefore y_c = .4375 mm$ (downward) Ans

Q.2 एक समान भार की और 3m लम्बी एक कैंटीलीवर धरन पर दो बिंदु भार प्रयुक्त हैं। उनमें एक भार 20N का आबद्ध सिरे से 2m की दूरी पर और दूसरा भार 10N का मुक्त सिरे पर है मुक्त सिरे पर धरन का विक्षेप ज्ञात करो।

$E = 2\times10^5 \, N/mm^2$ एवं $I = \frac{550}{6}\times10^4 \, mm^4$

Sol:- Given:-

$l = 3m$

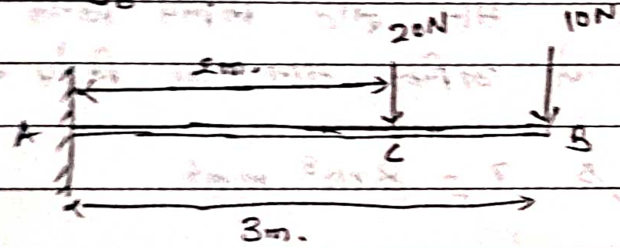
$W_1 = 20N$

$W_2 = 10N$

$E = 2\times10^5 \, N/mm^2 = 2\times10^{11} \, N/m^2$

$I = \frac{550}{6}\times10^4 \, mm^4 = \frac{550}{6}\times10^{-8} \, m^4$

$y_B = ?$



deflection at free end = Deflection at free end due to 20N + deflection at free end due to 10N

$= y_1 + y_2$

$\therefore y_1$ = deflection at free end due to 20N $= \frac{W_1 a^3}{3EI} + \frac{(l-a)\times W_1 a^2}{2EI}$

$$= \frac{20\times2^3}{3\times2\times10^{11}\times\frac{550}{6}\times10^{-8}} + \frac{(3-2)\times20\times2^2}{2\times2\times10^{11}\times\frac{550}{6}\times10^{-8}}$$

$y_1 = 51\times10^{-4} \, m.$

deflection at free end due to 10N $= \frac{Wl^3}{3EI}$

$$= \frac{10 \times 2^3}{3 \times 2 \times 10^{11} \times \frac{550}{6} \times 10^{-8}}$$

$$= 4.9 \times 10^{-4} \text{ m.}$$

$\therefore$ total deflection at free end $y_B = 5.1 \times 10^{-4} + 4.9 \times 10^{-4}$

$$= 10^{-3} \text{ m. Ans}$$

Q:3 4m लम्बी एक कैंटी उत्तोलक धरन के स्वतंत्र सिरे पर 2W N एवं मध्य बिन्दु पर W N का संकेन्द्रीत भार प्रयुक्त ... W का वह अधिकतम मान ज्ञात किजिये जिसके क्रियाशील होने पर धरन में 3.7 m.m. से अधिक मान का विक्षेप न होने पाये

$E = 2 \times 10^5 \ N/mm^2$ & $I = 8 \times 10^8 \ m.m^4$

Sol: Given'-

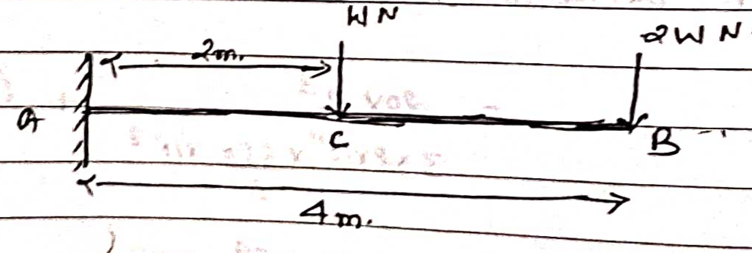
$W_1 = 2W$ N

$W_2 = W$ N.

$y_{max} = 3.7 m.m. = 3.7 \times 10^{-3}$ m.

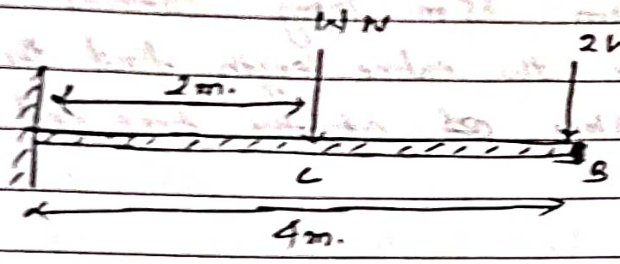
$E = 2 \times 10^5 \ N/mm^2 = 2 \times 10^{11} \ N/m^2$

$I = 8 \times 10^8 \ mm^4 = 8 \times 10^{-4} \ m^4$ $\quad W = ?$



max deflection will be at free end so

$\therefore \quad y_B = y_{max} = 3.7\times10^{-3}$



deflection at point B $= y_B = y_1 + y_2$

$y_1 =$ deflection at free end due to W N load

$$\Rightarrow \frac{W \times a^3}{3EI} + (L-a)\frac{Wa^2}{2EI}$$

$$= \frac{W \times 2^3}{3\times2\times10^{11}\times8\times10^{-4}} + (4-2)\frac{W\times2^2}{2\times2\times10^{11}\times8\times10^{-4}}$$

$$y_1 = 41.67\times10^{-9}\,W.$$

$y_2 =$ deflection at free end due to 2W N.

$$= \frac{2W\times4^3}{3\times2\times10^{11}\times8\times10^{-4}}$$

$$y_2 = 266.67\times10^{-9}\,W$$

$\therefore$ total deflection $= y_B = 41.67\times10^{-9}W + 266.67\times10^{-9}W$

$$3.7\times10^{-3} = 308.3\times10^{-9}\,W$$

$$W = 11999.87\text{ N} \approx 12000\text{ N}$$

$W = 12000$ N. Ans

Q:4 A cantilever beam 1.5 m. long carries a uniformaly distributed load over the entire length find the deflection at the free end if the slope at the free end is 1.5°.

Sol: Given

$l = 1.5$ m.

$\theta = 1.5° = \frac{1.5 \times \pi}{180}$ rad.

$y = ?$

$\therefore$ deflection at free end $= y = \frac{W l^3}{8EI}$

$= \frac{W l^3 \times l}{E \times I \times 8}$ — ①

and $\theta = \frac{1.5 \times \pi}{180} = \frac{W l^3}{6 \times EI}$

$\therefore \frac{W l^3}{EI} = \frac{1.5 \times 6 \times \pi}{180}$

put this value in eq. ①

$\therefore$

$y = 1.5 \times 6 \times \frac{\pi}{180} \times \frac{1.5}{8}$

$= 29.45 \times 10^{-3}$ m. Ans

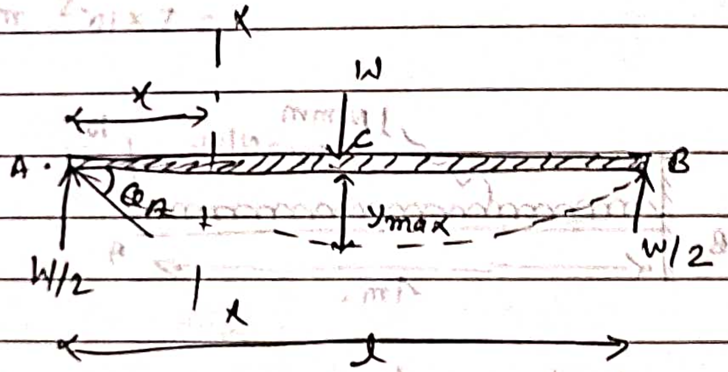
$y_2 = 3.2 \times 10^{-3}$ m.

$\therefore$ deflection at free end $= y = 21.33 \times 10^{-6} + 3.2 \times 10^{-3}$

$y_A = 3.221 \times 10^{-3}$ m. Ans

II Simply supported beam :-

Case I :- Simply supported beam of span l carrying a point load at mid span.



A simply supported beam AB length l having a point load W at mid point C.

$\because$ load is acting in mid point so max. deflection will occur at mid point span.

Let's take a section X-X at a distance x from point A.

So Bending moment at any section XX is given by -

$$BM_x = M_x = \frac{W}{2}x$$

$$\therefore EI\frac{d^2y}{dx^2} = M_x$$

$$\therefore \quad EI\frac{d^2y}{dx^2} = +\frac{W}{2}x$$

Integrating both side we get-

$$EI\frac{dy}{dx} = \frac{W}{4}x^2 + c_1$$

at $x = l/2$ $\quad \frac{dy}{dx} = 0$

$$\therefore \quad 0 = \frac{W}{4} \times \frac{l^2}{4} + c_1$$

$$c_1 = -\frac{Wl^2}{16}$$

$$\therefore \quad EI\frac{dy}{dx} = \frac{Wx^2}{4} - \frac{Wl^2}{16} \quad \text{—— (I)}$$

again on integrating both side

$$EIy = \frac{Wx^3}{12} - \frac{Wl^2}{16}x + c_2$$

at $x = 0$ $\quad y = 0$

$$0 = 0 - 0 + c_2$$

$$\therefore c_2 = 0.$$

$$\therefore \quad EIy = \frac{Wx^3}{12} - \frac{Wl^2x}{16} \quad \text{—— (II)}$$

$\therefore$ Slope will be given by put $x = 0$ at eq. (I)

$$EI\frac{dy}{dx} = 0 - \frac{Wl^2}{16}$$

$$\boxed{\frac{dy}{dx} = \theta_A = -\frac{Wl^2}{16EI}}$$

and max. deflection will be given by put $x = l/2$ at eq. (11).

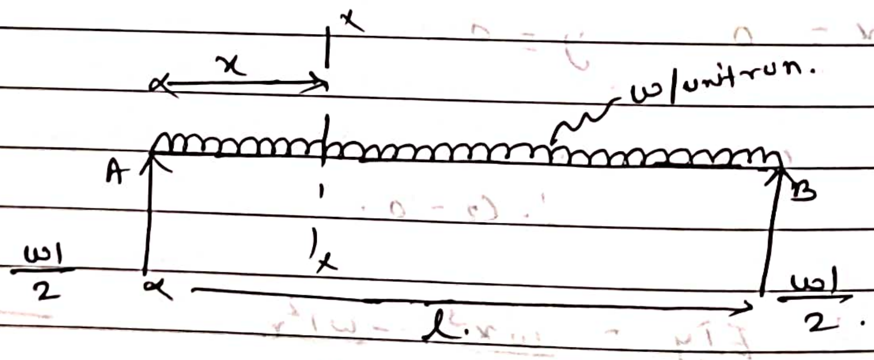
$$EI\, y_{max} = \frac{w}{12} \times \left(\frac{l}{2}\right)^3 - \frac{wl^2}{16}$$

$$= \frac{wl^3}{96} - \frac{wl^3}{32}$$

$$= \frac{wl^3}{48}$$

$$\therefore \boxed{y_{max} = \frac{Wl^3}{48EI}} \quad \underline{Ans}$$

Case-II simply supported beam carrying a udl of w per unit run over the whole span -



Simply supported beam of length l carrying a udl w per unit run over the whole span.

let's take a section xx at a distance x from point A.

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taking moment along section -

$$M_x = \frac{wl \times x}{2} - \frac{wx^2}{2}$$

$\because$ $M_x = \frac{d^2y}{dx^2} \times EI$

$\therefore$ $$EI \frac{d^2y}{dx^2} = \frac{wlx}{2} - \frac{wx^2}{2}$$

Integrating both side

$$EI \frac{dy}{dx} = \frac{wlx^2}{4} - \frac{wx^3}{6} + c_1 \quad \text{---} (i)$$

the max deflection occur at mid span so slope at mid span equals zero.

$\therefore$ at $x = l/2$ $\frac{dy}{dx} = 0$

$$0 = \frac{wl}{4} \times \frac{l^2}{4} - \frac{w}{6} \times \frac{l^3}{8} + c_1$$

$$= \frac{wl^3}{16} - \frac{wl^3}{48} + c_1$$

$$c_1 = \frac{-wl^3}{24}$$ put the value in (i)

$\therefore$ $$EI \frac{dy}{dx} = \frac{wlx^2}{4} - \frac{wx^3}{6} - \frac{wl^3}{24}$$ ---- Slope eqn.

$\therefore$ ~~stop~~ slope at point A will be given by -

at $x = 0$

$$EI \left(\frac{dy}{dx}\right)_A = 0 - 0 - \frac{wl^3}{24}$$

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$$\therefore \boxed{\theta_A = \frac{dy}{dx} = \frac{-wl^3}{24EI}}$$

Integrating the slope equation

$$EIy = \frac{wl}{12}x^3 - \frac{wx^4}{24} - \frac{wl^3x}{24} + C_2.$$

at $x = 0$ $y = 0$ $\therefore C_2 = 0$

$$\therefore EIy = \frac{wlx^3}{12} - \frac{wx^4}{24} - \frac{wl^3x}{24}.$$

max deflection occur at mid span so put $x = l/2$

$$EIy = \frac{wl}{12}\left(\frac{l}{2}\right)^3 - \frac{w(l/2)^4}{24} - \frac{wl^3}{24} \times \frac{l}{2}.$$

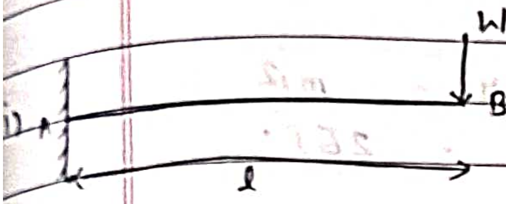
$$y_{max} = \frac{5wl^4}{384EI}$$

$$\therefore \boxed{y_{max} = \frac{5wl^4}{384EI}} \quad Ans$$

Important tables :-

Condition | formulae.

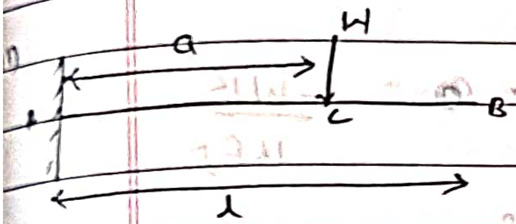
(i)



slope $= \dfrac{-Wl^2}{2EI}$

deflection $y_B = \dfrac{-Wl^3}{3EI}$

(ii)



slope $= \theta_A = \theta_B = \dfrac{-Wa^2}{2EI}$

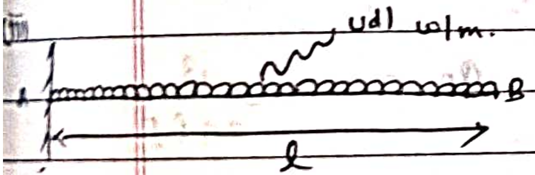
deflection :-

$y_c = \dfrac{-Wa^3}{3EI}$

$y_B = y_c + (l-a)\,\theta_B$

$= \dfrac{-Wa^3}{3EI} + (l-a) \times - \dfrac{Wa^2}{2EI}$

(iii)



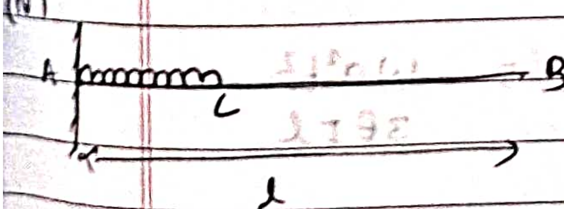
slope =

$\theta_B = \dfrac{-wl^3}{6EI}$

deflection

$y_B = \dfrac{-wl^4}{8EI}$.

(iv)



slope :-

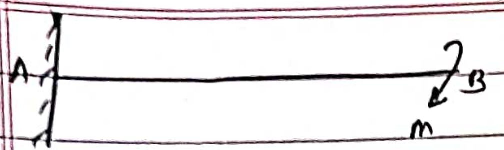
$\theta_B = \theta_C = \dfrac{-wa^3}{6EI}$

deflection.

$y_c = \dfrac{-wa^4}{8EI}$

$y_B = y_c + (l-a)\,\theta_B$.

(V)



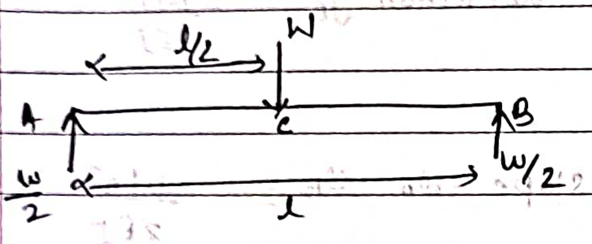
Slope :-

$$\theta_B = -\frac{Ml}{EI}$$

deflection :

$$y_B = -\frac{Ml^2}{2EI}$$

(VI)



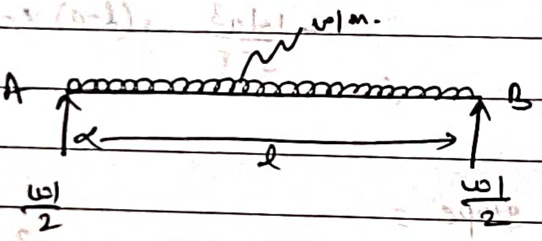
slope

$$\theta_A = \theta_B = -\frac{Wl^2}{16EI}$$

deflection

$$y_C = \frac{-Wl^3}{48EI}$$

VII



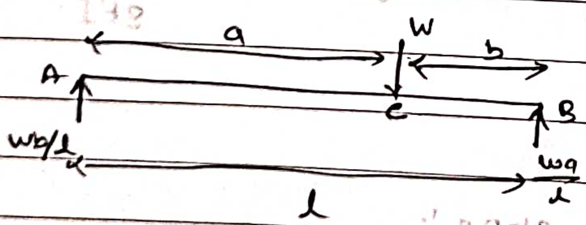
Slope :-

$$\theta_A = -\frac{wl^3}{24EI}$$

deflection

$$y_{max} = -\frac{5wl^4}{384EI}$$

VIII



deflection :-

$$y_C = -\frac{Wa^2b^2}{3EIl}$$

$$y_{max} = -\frac{Wb(l^2-b^2)^{3/2}}{9\sqrt{3}\,EIl}$$

Position of max. deflection

$x = \sqrt{l^2 - b^2}$

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Q:1 ~~2m लम्बी एक कैंटी उत्तोलक धरन के मुक्त सिरे पर 10KN का भार प्रयुक्त है~~

Q:1 एक शुद्धालम्बित धरन की विस्तृति 4m. है। उस के मध्य बिंदु पर एक संकेन्द्रित भार प्रयुक्त है। यदि धरन के सिरों पर अधिकतम ढाल 1° हो तो इस दशा में अधिकतम विक्षेप कितना होगा।

Sol: Given:

$l = 4m.$

$\theta_A = 1°$

$\because 180° = \pi$ radian

$\therefore 1° = \frac{\pi}{180}$ radian.

$$\theta = \frac{Wl^2}{16EI} = \frac{\pi}{180}$$

and

$$y_{max} = \frac{Wl^3}{48EI} = \frac{Wl^2}{16EI} \times \frac{l}{3}$$

$$= \frac{\pi}{180} \times \frac{4}{3} = 23.7 \times 10^{-3} \text{ m. Ans}$$

Q:2 8m. लम्बी शुद्धालम्बित धरन की समस्त लम्बाई पर 10KN/m का समवितरित भार प्रयुक्त है। यदि $E \times I = 4000$ KN-m2 हो तो अधिकतम ढाल विक्षेप ज्ञात करो।

Sol: Given:-

$l = 8m.$

$w = 10 \times 10^3$ N/m.

$E \times I = 4000 \times 10^3$ N-m2

$\theta = ?$

$y_{max} = ?$

$$\theta_1 = \frac{wl^3}{24EI} = \frac{10\times10^3 \times 8^3}{24 \times 4000\times10^3}$$

$$= .0533 \text{ radian.}$$

and deflection

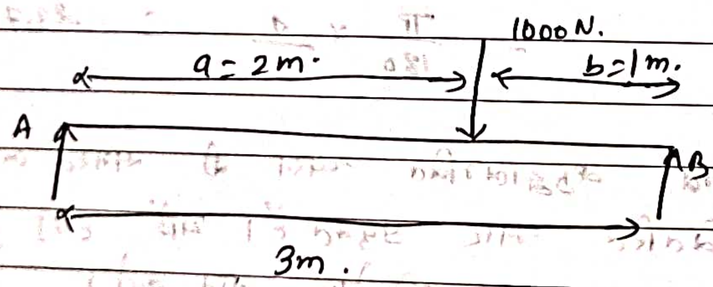
$$y_{max} = \frac{5wl^4}{384EI}$$

$$= \frac{5 \times 10\times10^3 \times 8^4}{384 \times 4000\times10^3}$$

$$y_{max} = .133 \text{ m.}$$ Ans

Q:-3 3m. विस्तृति की एक शुद्धालम्ब धरन पर बांयी आलम्ब से 2m. की दूरी पर एक संकेन्द्रित भार जिसका मान 1000N है क्रियाशील है। अधिकतम विक्षेप का मान ज्ञात करो जब $E = 2\times10^5$ N/mm² $I = 3\times10^7$ mm⁴ हो।

Sol:-



Given :-
$W = 1000$ N.
$a = 2$ m. $b = 1$ m.
$l = 3$ m.
$E = 2\times10^5$ N/mm²
$= 2\times10^{11}$ N/m²
$I = 3\times10^7$ mm⁴
$= 3\times10^{-5}$ m⁴

max. deflection occurs at

$$x = \sqrt{\frac{l^2-b^2}{3}}$$

$$= \sqrt{\frac{3^2-1^2}{3}} = 1.63 \text{ m. from A.}$$

$\therefore$

$$y_{max} = \frac{W b (l^2-b^2)^{3/2}}{9\sqrt{3} E I l}$$

$$= \frac{1000 \times 1 \times (3^2-1^2)^{3/2}}{9\sqrt{3} \times 2\times10^{11} \times 3\times10^{-5} \times 3}$$

$$y_{max} = 80 \times 10^{-6} \text{ m.} \quad \underline{Ans}$$

Unit - 7

Buckling stresses in collomn & strut types components

Important defination :-

(i) strut :- A bar or a member of a structure in any position and carrying an axial compressive load is called a strut.

(ii) Column :- A column is a long vertical selender bar or vertical member, subjected to an axial compressive load.

(iii) Slenderness ratio :- the ratio of the length of the column to the least radius of gyration is called Slenderness ratio.

(iv) Buckling factor :- It is the ratio between equivalent length of the column to the minimum radius of gyration.

(v) Buckling load :- the minimum axial load at which the column tends to have lateral displacement or buckle is called the buckling or crippling or critical load.

(vi) Safe load :- It is the load to which a column is actually subjected to and it is well below the buckling load.

$$\text{Safe load} = \frac{\text{buckling load}}{\text{Factor of safety}}$$

Classification of column :-

(i) Short Columns :- Column which have lengths less than 8 times to their respective diameters or slenderness ratio is less than 32 are called short columns.

$$l < 8d \quad , \quad \text{slenderness ratio} < 32$$

(ii) Medium Columns :- Columns which have their length varying from 8 times their diameter to 30 times their diameter or their slenderness ratio lying between 32 and 120.

$$30d > l > 8d \quad \text{or} \quad \text{slenderness ratio} > 32, < 120$$

(iii) long column :- The column having their length more than 30 time to their diameter or slenderness ratio is greater than 120 known as long column.

$$l > 30d \quad \text{or} \quad \text{slenderness ratio} > 120.$$

* Strength of the column depends on selenderness ratio as selenderness ratio increased compressive strength of the column decrease and tendency to buckle increased.

Equivalent length (le) :- The equivalent length is defined as the distance between two adjacent points of contraflexure on the column.

The point of contraclassification of column :-

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* Strength of the column depends on selenderness ratio as selenderness ratio increased compressive strength of the column decrease and tendency to buckle increased.

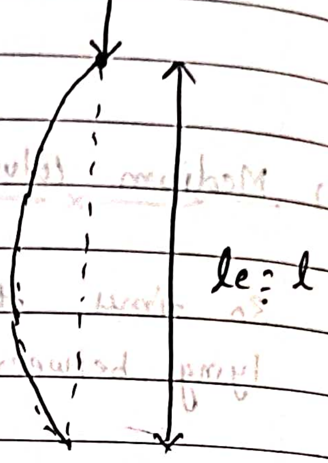
Equivalent length (le) :- the equivalent length is defined as the distance between two adjacent points of contraflexure on the column.

the point of contraflexure is defined as the point on the column where there is a change in the direction of the axis of the column

End conditions of the column :- there is 4 types of end con

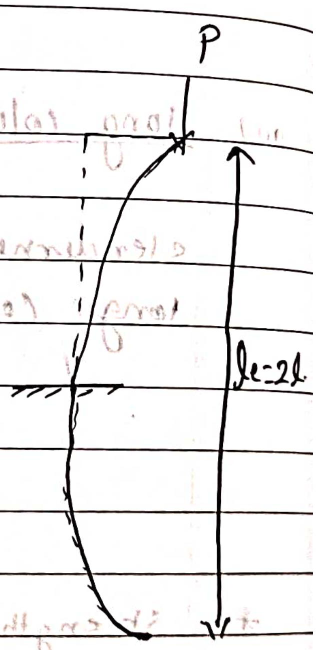
(i) Both end hinged or pin jointed :-

$$l_e = l$$



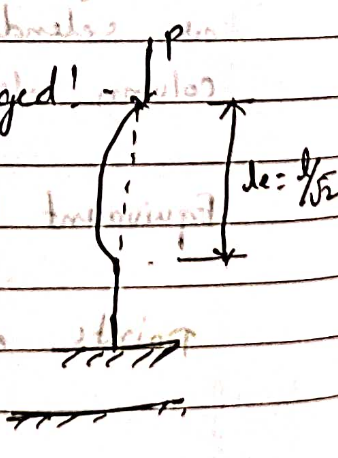
(ii) One end fixed other end free :-

$$l_e = 2l$$



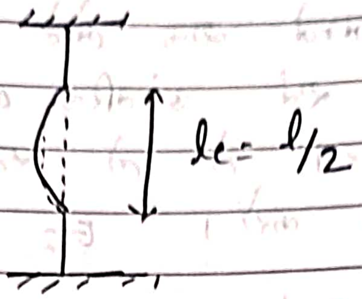
(iii) One end fixed other end pin jointed or hinged :-

$$l_e = l/\sqrt{2}$$



(iv) Both end fixed:-

$l_e = l/2$



Euler's theory:-

Assumption:-

(i) the column is intially straight and uniform lateral dimension.
(ii) the compressive load is exactly axial.
(iii) the material is homogeneous and isotropic.
(iv) the weight of column is neglected.
(v) limit of proportionality is not exceeded.

Formula:-

$$P_{euler} = \frac{\pi^2 \times E \times I_{min}}{l_e^2}$$

Where P_e = Critical load

E = Modulus of elasticity
l_e = equivalent length.
I_{min} = least Moment of Inertia of section

Limitation of Euler's formula:-

(i) Euler's formula is applicable to column which are intially exactly straight and load is axial.
(ii) this formula does not take account of the axial stress, thus the buckling load given by this formula may be much more than the actual buckling load.

Numericals:-

Q:-1 6m. लम्बे एक लोहे के खोखले स्तम्भ का बाह्य व्यास 200mm. एवं आंतरिक व्यास 180mm. है। यदि स्तम्भ के दोनों सिरे हिन्ज हो तो स्तम्भ पर प्रयुक्त अक्षीय सुरक्षित भार ज्ञात करें। $E = 1\times10^5$ N/mm2 FOS = 5

Sol:- Given that

$l = 6m.$

$D_o = 200mm.$

$D_i = 180mm.$

$E = 1\times10^5 \; N/mm^2 = 1\times10^{11} \; N/m^2$

$F.O.S. = 5$

$\because$ End condn = Both end hinged

so $l_e = l = 6m.$

$\therefore P_{euler} = \dfrac{\pi^2 EI}{l_e^2}.$

$I = \dfrac{\pi}{64}(D_o^4 - D_i^4)$

$= \dfrac{\pi}{64} \times (.2^4 - .18^4) = 27 \times 10^{-6} \; m^4.$

$\therefore P_{euler} = \dfrac{\pi^2 \times 1\times10^{11} \times 27\times10^{-6}}{6^2}$

$= 740.22 \times 10^3 \; N.$

$\therefore$ Safe load $= \dfrac{P_{euler}}{F.O.S}$

$= 148 \; KN.$ Ans

~~एक स्लेण्डर और एक ठोस स्तम्भ के सामर्थ्य का अनुपात ज्ञात करो जबकि दोनों के अनुप्रस्थ का क्षेत्रफल समान है~~

Q.2 A bar of length 4m. when used as a simply supported beam and subjected to a udl of 30KN/m. over the whole span, deflect 15m.m. at centre. Determine the crippling load when it is used as column with following end cond^n

(i) Both end pin jointed

(ii) one end fixed other ~~o~~ hinged

(iii) Both end fixed.

Sol:- Given :-

$l = 4m.$

$w = 30 KN/m.$

$\delta = 15 m.m. = .015m.$

∵ for simply supported beam with udl over the whole span deflection will be given by

$$\delta = \frac{5wl^4}{384EI}$$

$$EI = \frac{5 \times 30 \times 10^3 \times 4^4}{384 \times 15 \times 10^{-3}}$$

$$= \cancel{6.67 \times 10} \quad 6.67 \times 10^6 \; N\text{-}m$$

So

euler load will be

(i) Both end hinged :- $le = l$

$$P_{euler} = \frac{\pi^2 \times \cancel{6.67} EI}{le^2}$$

$$= \frac{\pi^2 \times 6.67 \times 10^6}{4^2} = 4114.3 \; KN.$$

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(ii) One end fixed other end ~~free~~ hinges

$\therefore l_e = 2l \cdot 8 \frac{1}{\sqrt{2}} = 2.83$

$\therefore P_{euler} = \frac{\pi^2 \times 6.67 \times 10^6}{2.83^2}$

$= 8219.6$ KN.

(iii) Both end fixed :- $l_e = \frac{l}{2} = 2m.$

$\therefore P_{euler} = \frac{\pi^2 \times 6.67 \times 10^6}{2^2}$

$= 16.47 \times 10^6$ N. Ans

Q: Determine the ratio of the buckling strength of ~~the~~ two columns of circular cross-section one hollow and the solid when both are made of the same material, have the same length, cross-sectional area and end conditions. The internal diameter of the hollow ~~eoth~~ column is half of it's external diameter.

Sol:- Given :-

Area of Solid column = Area of hollow column

$A_S = A_H$

$\frac{\pi}{4} \times D_S^2 = \frac{\pi}{4}(D_H^2 - d_H^2)$

$\because d_H = \frac{D_H}{2}$ (Given)

$\frac{\pi}{4} \times D_S^2 = \frac{\pi}{4}\left(D_H^2 - \left(\frac{D_H}{2}\right)^2\right)$

$D_S^2 = D_H^2 - \frac{D_H^2}{4}$

$$D_s^2 = \frac{4D_H^2 - D_H^2}{4}$$

$$D_s^2 = \frac{3D_H^2}{4}$$

$$D_s = .866 D_H$$

$\therefore$ B

buckling strength of hollow column $- P_H = \frac{\pi^2 E I_H}{le^2}$

buckling strength of solid column $P_s = \frac{\pi^2 \times E \times I_s}{le^2}$

$$\therefore \frac{P_H}{P_s} = \frac{\cancel{\pi^2} \times \cancel{E}\, I_H}{\cancel{le^2}} \times \frac{\cancel{le^2}}{\cancel{\pi^2} \times \cancel{E} \times I_s}$$

$$= \frac{I_H}{I_s} = \frac{\cancel{\pi/64} \times (D_H^4 - d_H^4)}{\cancel{\pi/64} \times D_s^4}$$

$$= \frac{D_H^4 - \frac{D_H^4}{16}}{D_s^4} \qquad \frac{15/16\, D_H^4}{D_s^4}$$

$$= \frac{.9375\, D_H^4}{D_s^4}$$

$$= \frac{.9375 \times \cancel{D_H^4}}{(.866)^4 \times \cancel{D_H^4}}$$

$$= 1.66$$

$$\therefore \frac{P_H}{P_s} = 1.66 \quad \text{Ans}$$

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Rankine's Hypothesis for column:-

Rankine's hypothesis -

As per Rankine's hypothesis

$$\frac{1}{P_{Rankine}} = \frac{1}{P_c} + \frac{1}{P_e}$$

P_e = Euler load
P_c = crushing load -

On:-

$$\frac{1}{P_R} = \frac{1}{\sigma_c A} + \frac{1}{\pi^2 E A \left(\frac{k}{l_e}\right)^2}$$

On solving

$$P_{Rankine} = \frac{\sigma_c \times A}{1 + a\left(\frac{l_e}{k}\right)^2}$$

where:-
σ_c = crushing stress
A = Area
a = const.
l_e = equivalent length
k = Radius of gyration.

Numericals:-

Q: एक लम्बा लोहे का खोखला स्तम्भ, जिसके दोनों सिरे आबद्ध हैं, 5 m लम्बा है। उसका बाह्य व्यास 120 m.m. और धातु की मोटाई 15 m.m. है। यदि सम पीड़न प्रतिबल = 560 N/mm2 और रैंकीन स्थिरांक $a = \frac{1}{1600}$ हो तो स्तम्भ पर प्रयुक्त सुरक्षित अक्षीय भार ज्ञात कीजिये। सुरक्षा गुणांक = 5.

Given :- $l = 5m.$

$D = 120 m.m.$

$= .12 m.$

$t = 15 m.m.$

$= .015 m.$

$d = D - 2 \times t$

$= .09 m.$

$G_c = 560 N/mm2$

$= 560 \times 10^6 N/m2$

$a = \frac{1}{1600}$

$F.O.S. = 5$

$\therefore A = \frac{\pi}{4}(D^2 - d^2) = 4.94 \times 10^{-3} m^2$

$\cancel{P_R = \frac{G_c \times A}{1 + a\left(\frac{l_e}{K}\right)^2}}$

$M.O.I. = I = \frac{\pi}{64}(D^4 - d^4)$

$= 6.95 \times 10^{-6} m^4$

$= \frac{560 \times 10^6 \times 4.94 \times 10^{-3}}{1 + \frac{1}{1600}}$

$I = AK^2$

$K = .0375$

$\therefore$ both end fixed

so $l_e = \frac{l}{2} = 2.5m.$

$\therefore P_R = \frac{G_c \times A}{1 + a\left(\frac{l_e}{K}\right)^2} = \frac{560 \times 10^6 \times 4.94 \times 10^{-3}}{1 + \frac{1}{1600}\left(\frac{2.5}{.0375}\right)^2}$

$P_R = 732282.35$

$\therefore$ Safe load $= \frac{P_R}{F.O.S.} = 146456.4 N.$

$= 146.4 KN.$ Ans

Sol: Given:- $l = 5m.$

$D = 120 mm.$

$= .12m.$

$t = 15 mm.$

$= .015m.$

$d = D - 2\times t$

$= .09m.$

$\sigma_c = 560 N/mm^2$

$= 560\times10^6 N/m^2$

$a = \frac{1}{1600}$

F.O.S. = 5

$\therefore A = \frac{\pi}{4}(D^2-d^2) = 4.94\times10^{-3} m^2$

~~$P_R = \frac{\sigma_c \times A}{1+a\left(\frac{le}{K}\right)^2}$~~

M.O.I. $= I = \frac{\pi}{64}(D^4-d^4)$

$= 6.95\times10^{-6} m^4$

~~$= \frac{560\times10^6\times4.94\times10^{-3}}{1+\frac{1}{1600}}$~~

$I = AK^2$

$K = .0375$

$\because$ both end fixed

so $le = \frac{l}{2} = 2.5m.$

$\therefore$

$P_R = \frac{\sigma_c\times A}{1+a\left(\frac{le}{K}\right)^2} = \frac{560\times10^6\times4.94\times10^{-3}}{1+\frac{1}{1600}\left(\frac{2.5}{.0375}\right)^2}$

$P_R = 732282.35$

$\therefore$ Safe load $= \frac{P_R}{F.O.S.} = 146456.4 N.$

$= 146.4 KN.$ Ans

Q:- 3 m. लम्बे इस्पात की एक खोखली नली के लिए यूलर सूत्र एवं रैंकीन सूत्र से ~~प्रस्त~~ ~~प्रप्त~~ प्राप्त आंकुचन भार की तुलना करो। ~~एक~~ नली का बाह्य व्यास 50 m.m. और चद्दर की मोटाई 10 m.m. है दोनो सिरे हिन्ज है।

$E = 2\times10^5\ N/mm^2 \qquad \sigma_c = 320\ N/mm^2$ और $a = \frac{1}{7500}$

Sol:- Given

$l = 3m. \qquad E = 2\times10^{11} N/m^2 \qquad \sigma_c = 320\times10^6\ N/m^2$

$a = 1/7500$

$\because$ both end hinged $\therefore l_e = l$

$= 3m.$

$D = 50\ m.m. = .05m.$

$t = \cancel{.01}\ 10 m.m$

$\therefore d = D - 2t = .03 m.$

$$\text{Area} = A = \frac{\pi}{4} \times (D^2 - d^2)$$

$$= 1.256 \times 10^{-3}\ m^2$$

$$\text{Moment of Inertia} = I = \frac{\pi}{64} \times (D^4 - d^4)$$

$$= \frac{\pi}{64}(.05^4 - .03^4)$$

$$I = 2.67 \times 10^{-7}\ m^4$$

$\because I = K^2 \times A$

$$K = \sqrt{\frac{I}{A}} = .0146\ m$$

$\therefore$ Euler load $P_{euler} = \dfrac{\pi^2 \times E \times I}{le^2}$

$$= \frac{\pi^2 \times 2 \times 10^{11} \times 2.67 \times 10^{-7}}{3^2}$$

$$P_{euler} = 58559.6 \text{ N.}$$

and Rankine load $= \dfrac{\sigma_c \times A}{1 + a\left(\frac{le}{k}\right)^2}$

$$= \frac{320 \times 10^6 \times 1.256 \times 10^{-3}}{1 + \frac{1}{7500} \times \left(\frac{3}{.0146}\right)^2}$$

$$P_{Rankine} = \cancel{256472.4 \text{ N.}}$$
$$60625.31 \text{ N}$$

$$\therefore \frac{P_{euler}}{P_{Rankine}} = \frac{58559.6}{60625.31} = .96$$ Ans